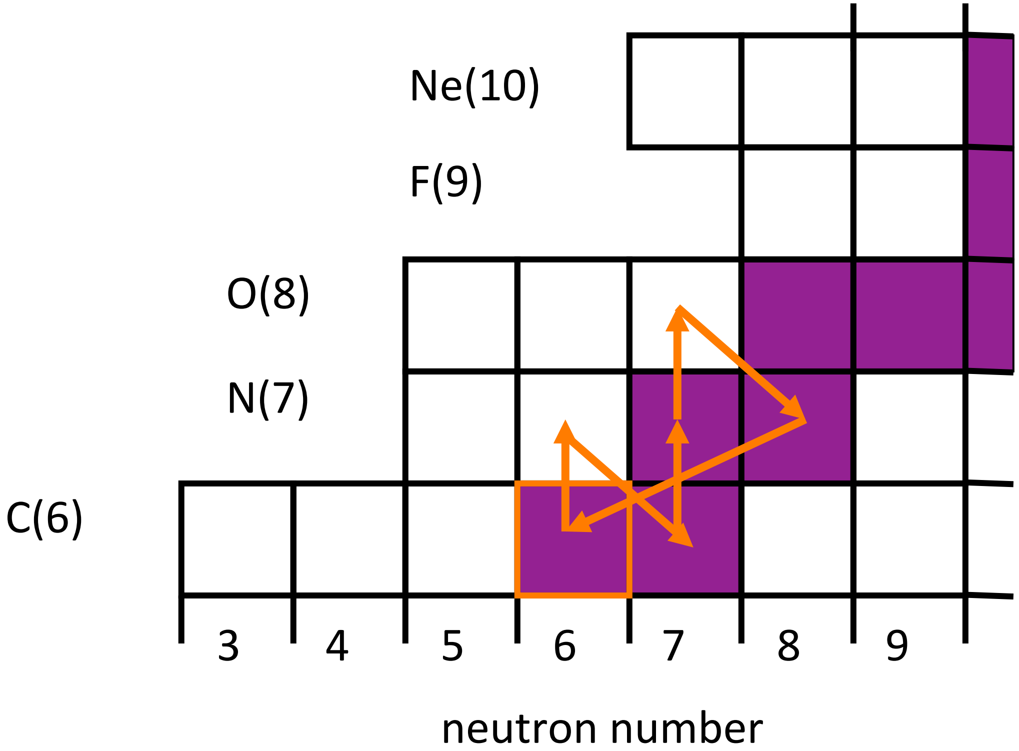
Reaction Network After the He^3 Equilibrium

$$\begin{aligned}\frac{d[H]}{dt} &= -\lambda_{11}[H]^2 - 2\lambda_{34}[He^3]_{eq}[He^4] \\ \frac{d[He^4]}{dt} &= \frac{1}{4}\lambda_{11}[H]^2 + \frac{1}{2}\lambda_{34}[He^3]_{eq}[He^4]\end{aligned}$$

$$\frac{d[He^4]}{dt} = -\frac{1}{4}\frac{d[H]}{dt}$$

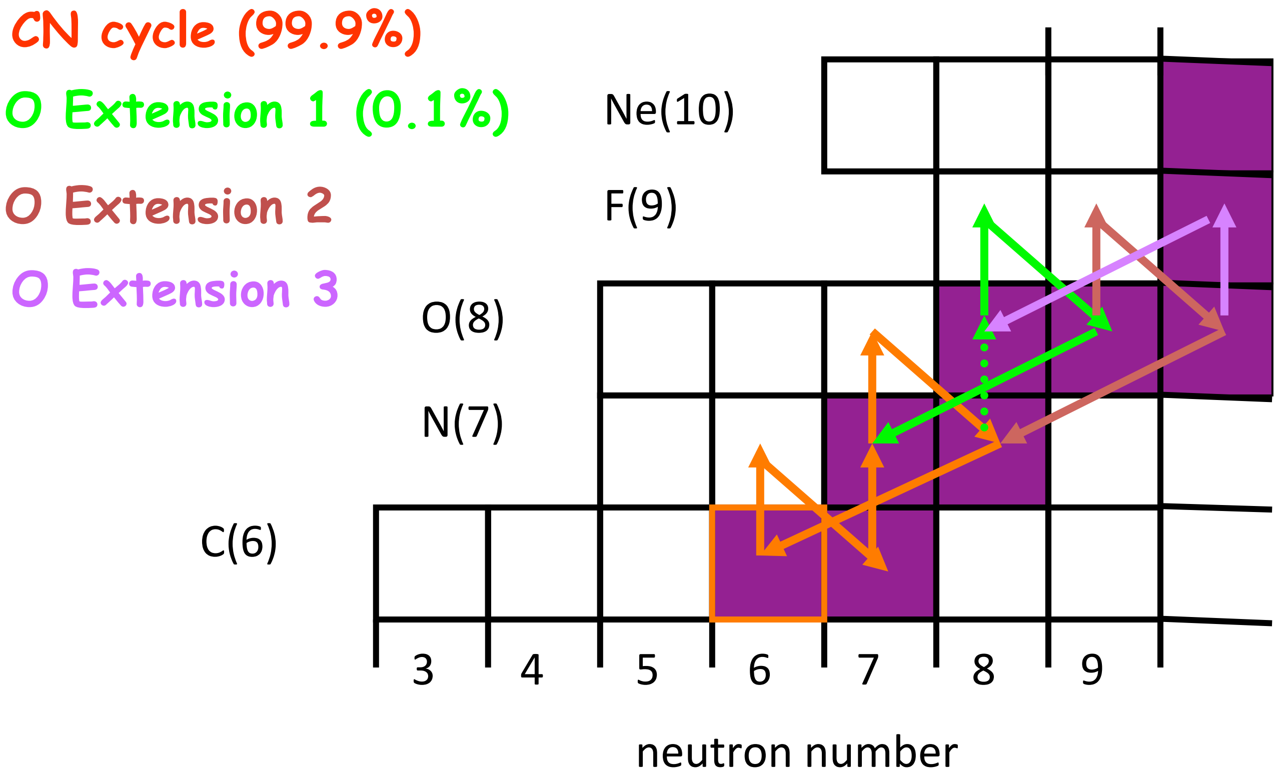
The CN cycle



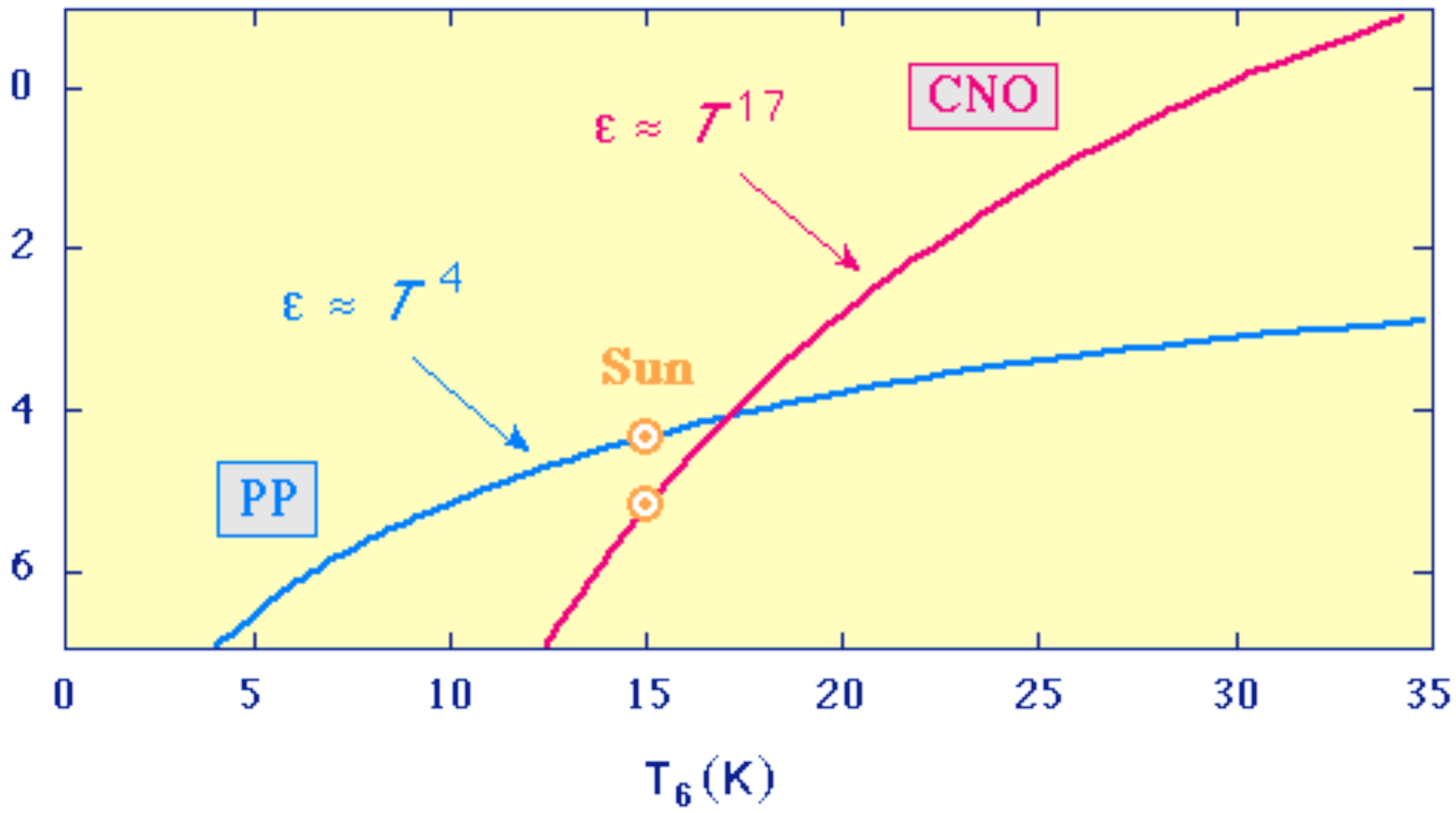
$^{12}C(p,\gamma)^{13}N\ \ (\beta^+)^{13}C\ \ (p,\gamma)^{14}N\ \ (p,\gamma)^{15}O\ \ (\beta^+)^{15}N\ (p,\alpha)^{12}C$

Net effect: $4p \rightarrow \alpha + 2e^+ + 2\nu_e$

CNO cycle


CN cycle (99.9%)
O Extension 1 (0.1%)
O Extension 2
O Extension 3
Ne(10)
F(9)
O(8)
N(7)
C(6)
3
4
5
6
7
8
9
neutron number

Competition between the p-p chain and the CNO Cycle



Helioseismology - Definitions of Characterizing Quantities

Static, stable star at spherically-symmetric equilibrium

- Pressure $p(r)$,
- Mass density $\rho(r)$,
- Gravitational potential $\phi(r)$,
- Rate of nuclear energy generation $\epsilon(r)$,
- Temperature $T(r)$,
- Energy flux $\mathbf{F}$,
- Entropy s.
- Adiabatic indices

$$\Gamma_1 = \left(\frac{\partial \log p}{\partial \log \rho}\right)_s \qquad \Gamma_3 - 1 = \left(\frac{\partial \log T}{\partial \log \rho}\right)_s,$$

- The total derivative $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$

A static star:

Equation of motion

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - \rho \nabla \phi$$

Poisson's equation for gravitational attraction

$$\nabla^2 \phi = 4\pi G \rho$$

Equation of continuity

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0$$

Energy conservation

$$\frac{1}{p}\frac{Dp}{Dt} - \Gamma_1 \frac{1}{\rho}\frac{D\rho}{Dt} = \frac{\Gamma_3 - 1}{p}(\rho\epsilon - \nabla \cdot \mathbf{F})$$

Stellar Seismology

Eulerian perturbations

$$\rho(\mathbf{r},t) = \rho_0(\mathbf{r}) + \rho'(\mathbf{r},t), \quad \mathbf{v} = \frac{\partial}{\partial t}(\delta\mathbf{r})$$

Conservation of momentum

$$\rho\frac{\partial^2\delta\mathbf{r}}{\partial t^2} = -\nabla p' + \frac{\rho'}{\rho}\nabla p - \rho\nabla\phi'$$

Equation of continuity

$$\rho' + \nabla\cdot(\rho\delta\mathbf{r}) = 0$$

Poisson's equation

$$\nabla^2\phi' = 4\pi G\rho'$$

Energy conservation

$$\frac{\rho'}{\rho} + \frac{1}{\rho}\delta\mathbf{r}\cdot\nabla\rho = \frac{1}{\Gamma_1}\left(\frac{p'}{p} + \frac{1}{p}\delta\mathbf{r}\cdot\nabla p\right)$$

Normal modes

periodic time dependence

$$\rho'(\mathbf{r},t) \sim \rho'(r) Y_{\ell m}(\theta,\phi)\exp(-i\omega t)$$

Adiabatic sound speed

$$c^2 = \frac{\Gamma_1 p}{\rho}$$

Stellar oscillations

$$\frac{d^2\Psi}{dr^2} + \frac{1}{c^2}\left[\omega^2 - \omega_{\mathrm{co}}^2 - \frac{\ell(\ell+1)c^2}{r^2}\left(1 - \frac{N^2}{\omega^2}\right)\right]\Psi \simeq 0$$

$$\boxed{\Psi(r) = c^2\rho^{1/2}\nabla\cdot\delta\mathbf{r}}$$

Buoyancy frequency

$$N^2 = \frac{Gm_0(r)}{r}\left(\frac{1}{\Gamma_1}\frac{d\log p}{dr} - \frac{d\log\rho}{dr}\right)$$

Acoustical cut-off freq.

$$\omega_{\mathrm{co}}^2 = \frac{c^2}{4H^2}\left(1 - 2\frac{dH}{dr}\right)$$

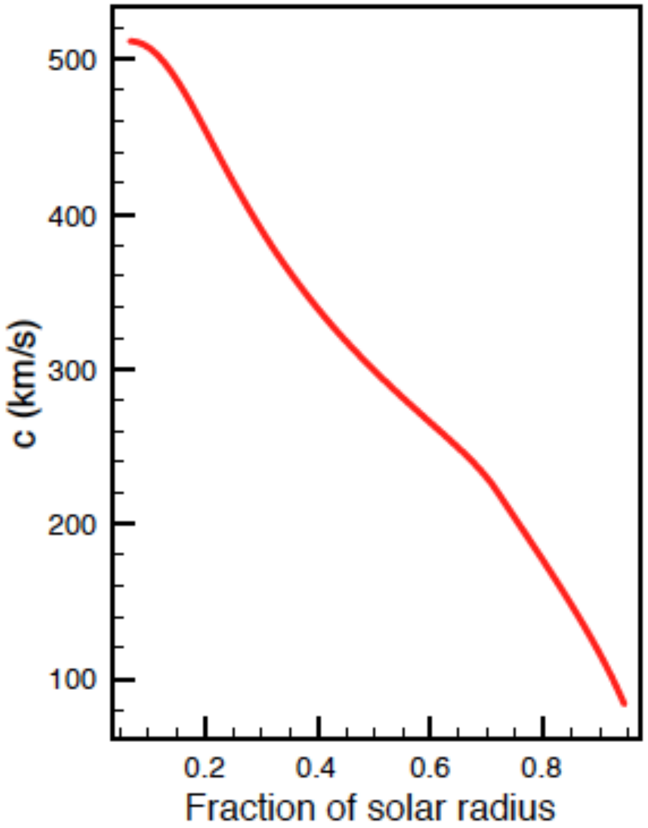
$$H = -(d\log\rho/dr)^{-1}$$

density scale height

p- and g-modes

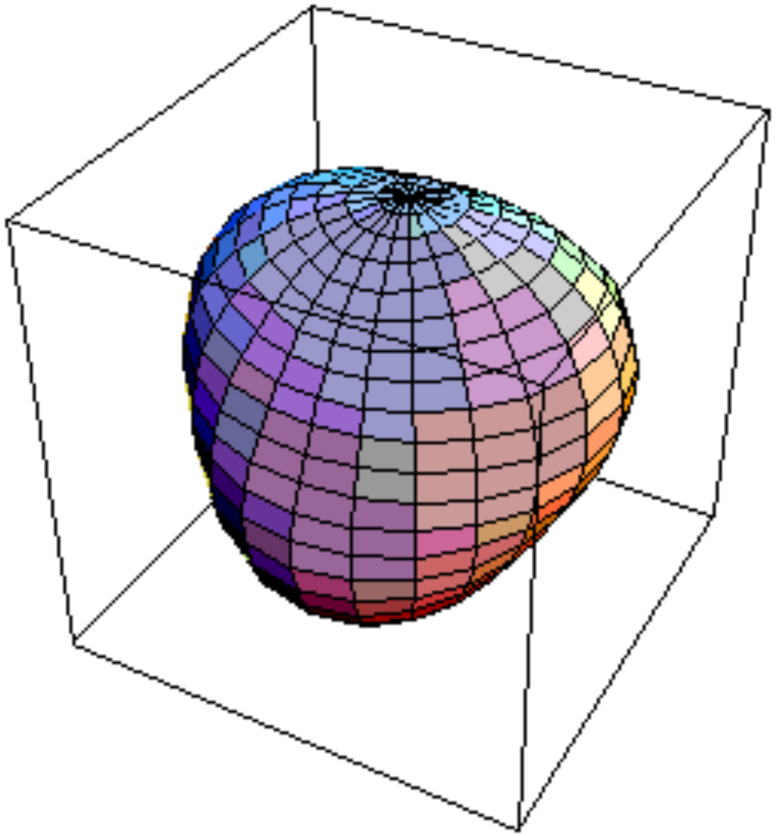
Stellar oscillations

$$\frac{d^2\Psi}{dr^2} + \frac{1}{c^2}\left[\omega^2 - \omega_{\rm co}^2 - \frac{\ell(\ell+1)c^2}{r^2}\left(1 - \frac{N^2}{\omega^2}\right)\right]\Psi \simeq 0$$



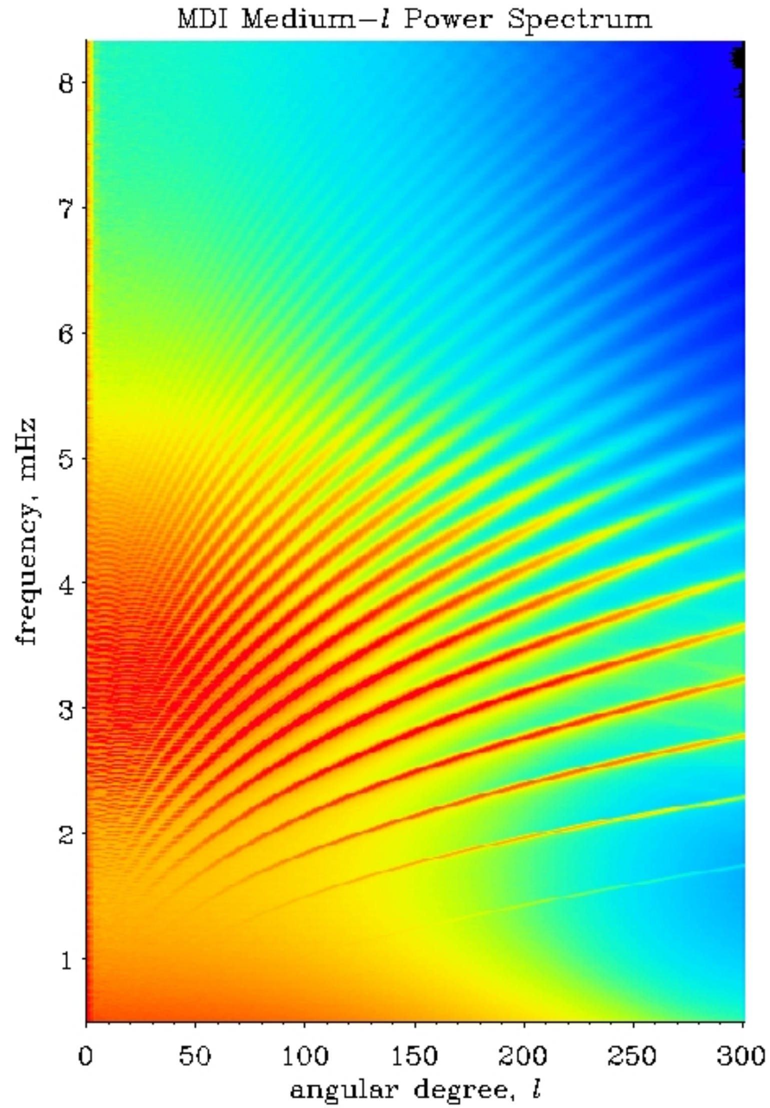
- N is $\sim$ constant in the radiative zone, but zero in the convective zone
- $\omega_{\rm co}$ is monotonically decreasing
- $N^2/\omega^2 \ll 1 \Rightarrow$ the oscillations die out in the radiative zone (p-modes)
- $\frac{\ell(\ell+1)c^2}{r^2\omega^2} \gg 1 \Rightarrow$ the oscillations die out in the convective zone (g-modes)

Example: L=3, m=2 p-mode



From D. Guenther

Single Dopplergram Minus 45 Images Average
(30-MAR-96 19:54:00)
-500. -400. -300. -200. -100. 0. 100. 200. 300. 400. 500.
Velocity (m/s)
SOI / MDI
Stanford Lockheed Institute for Space Research

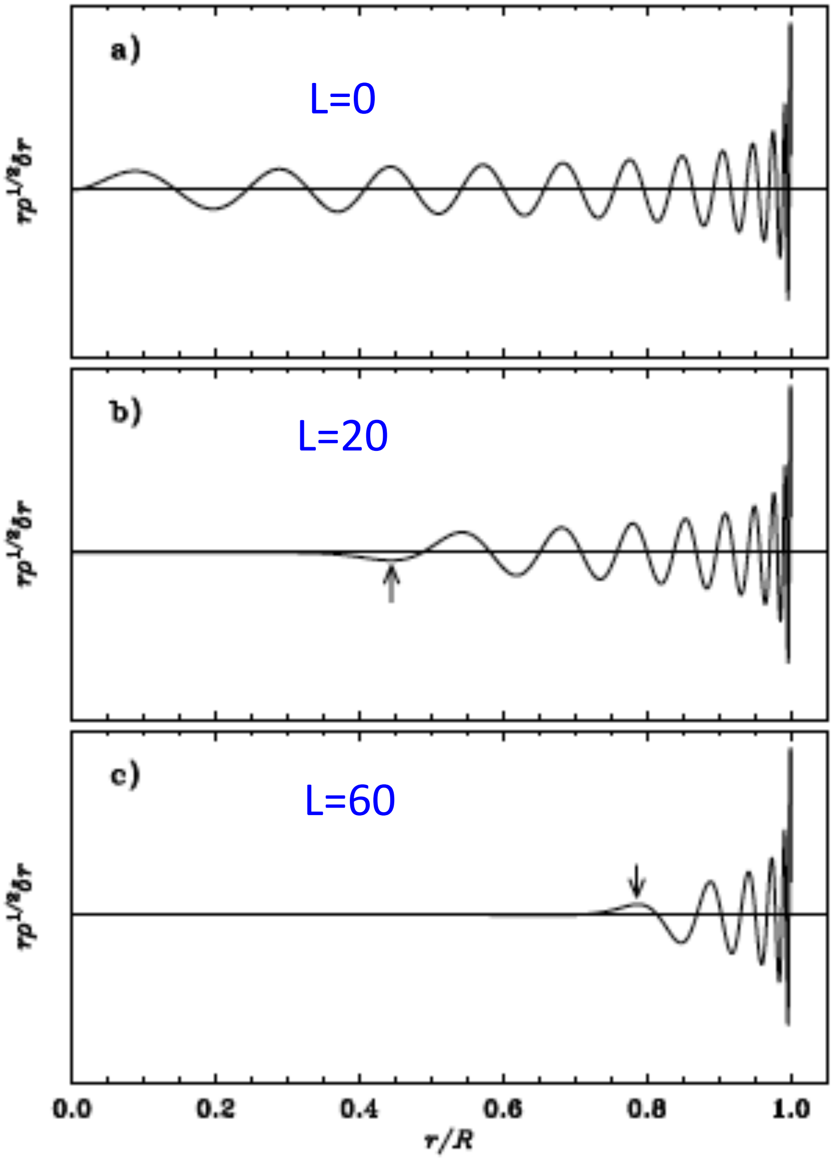


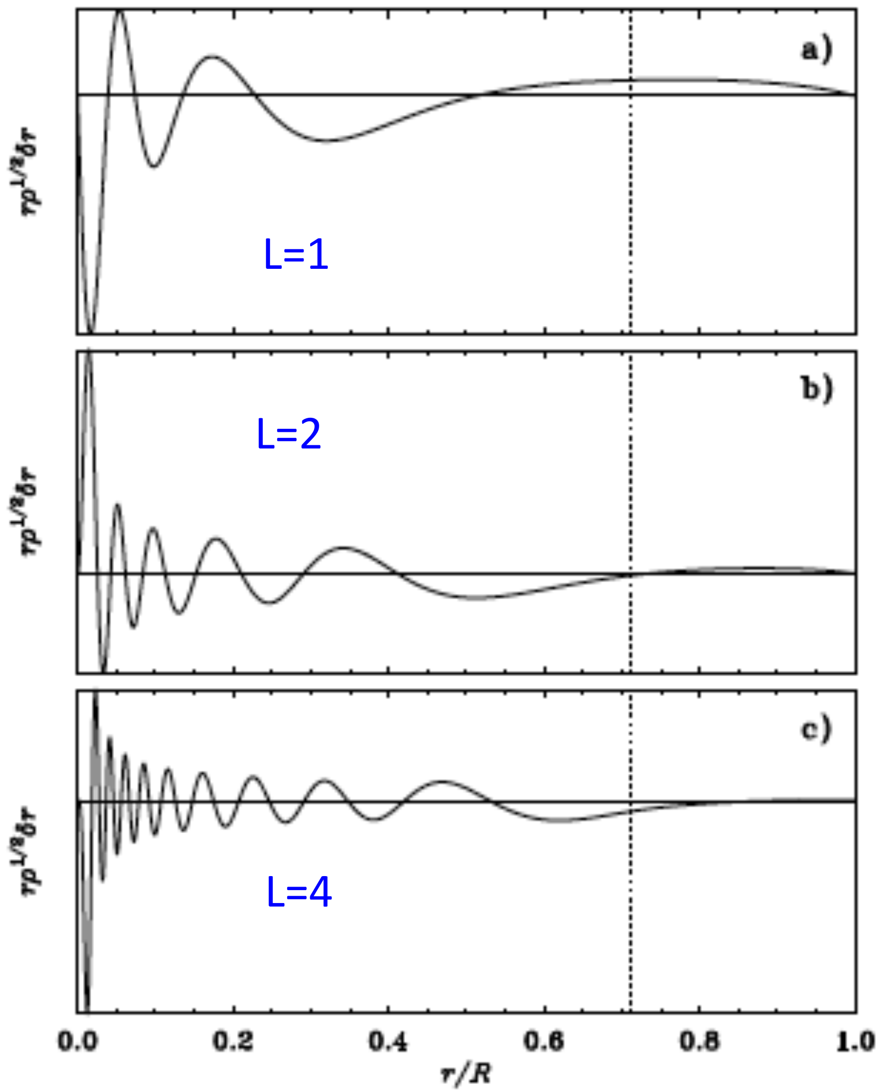
MDI Medium-l Power Spectrum
frequency, mHz
1
2
3
4
5
6
7
8
0 50 100 150 200 250 300
angular degree, l

SOHO data

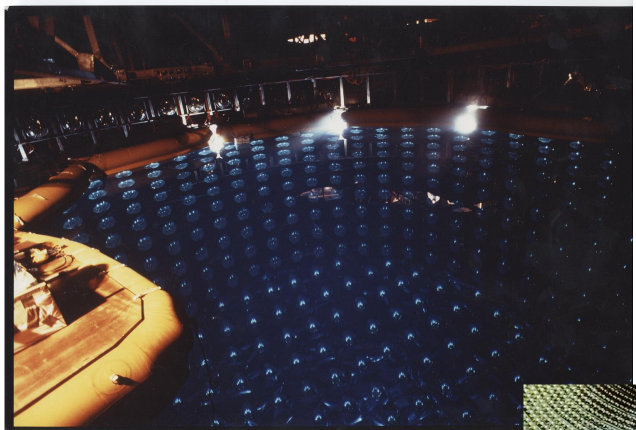
p-modes

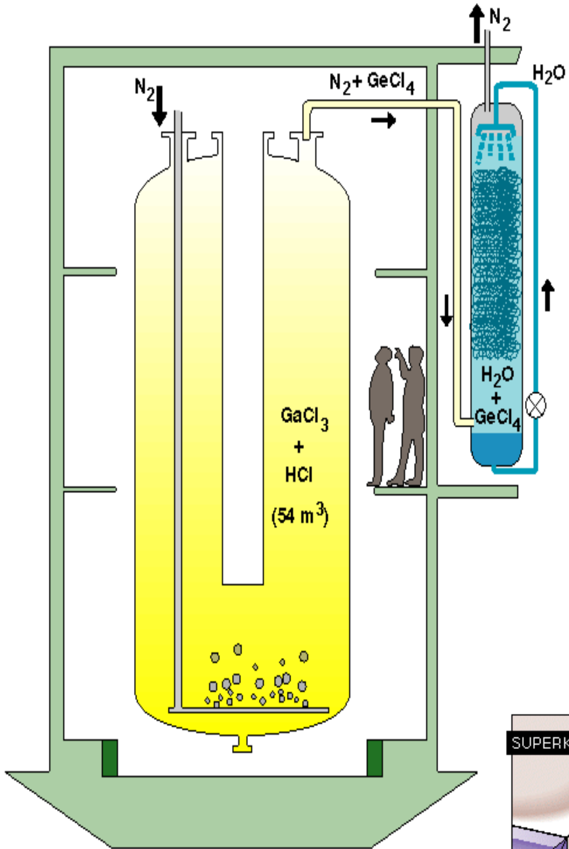
g-modes

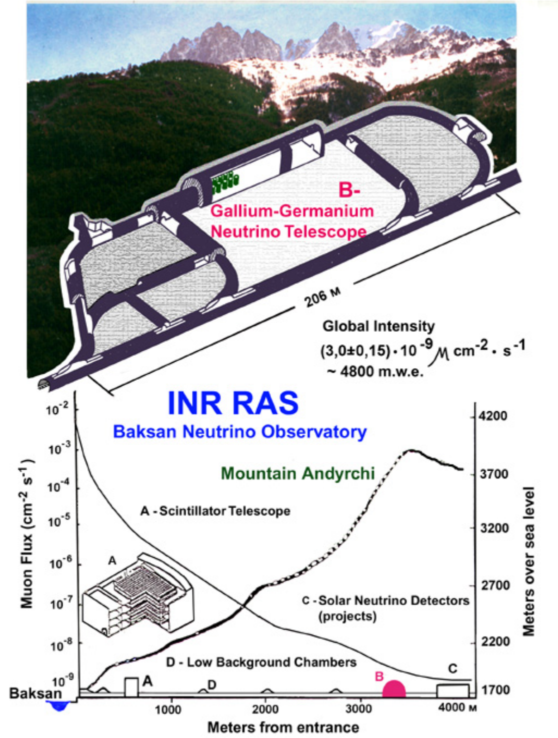




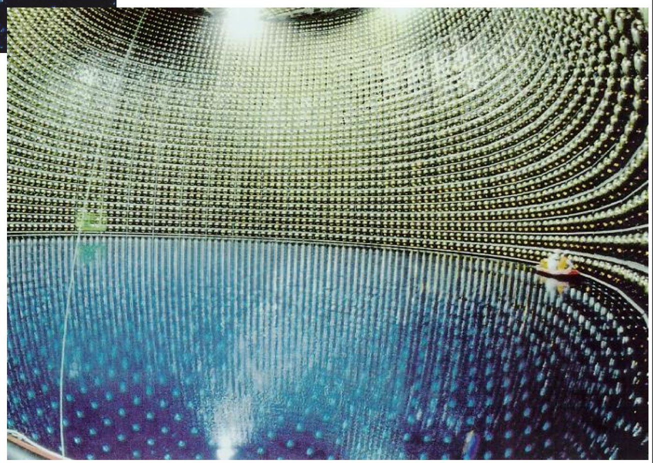
Christensen-Dalsgaard

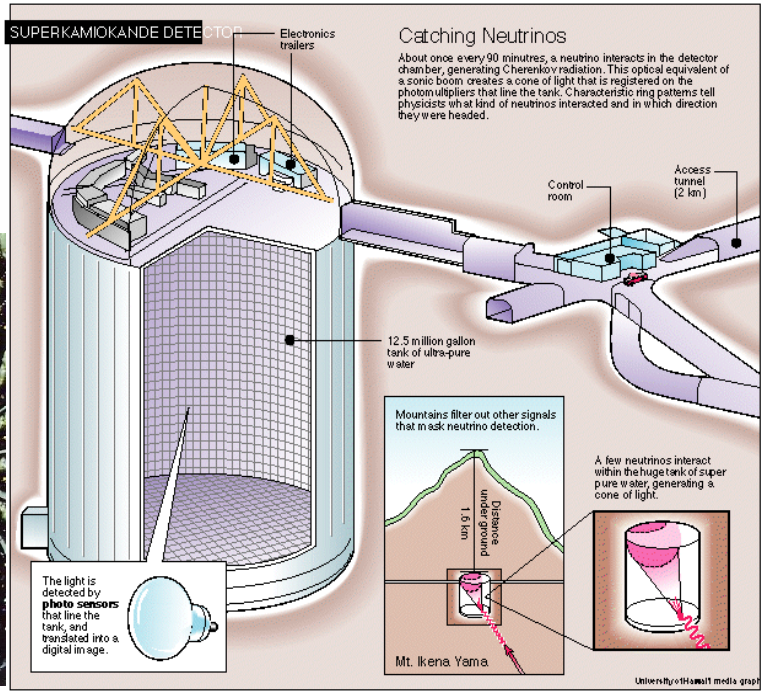




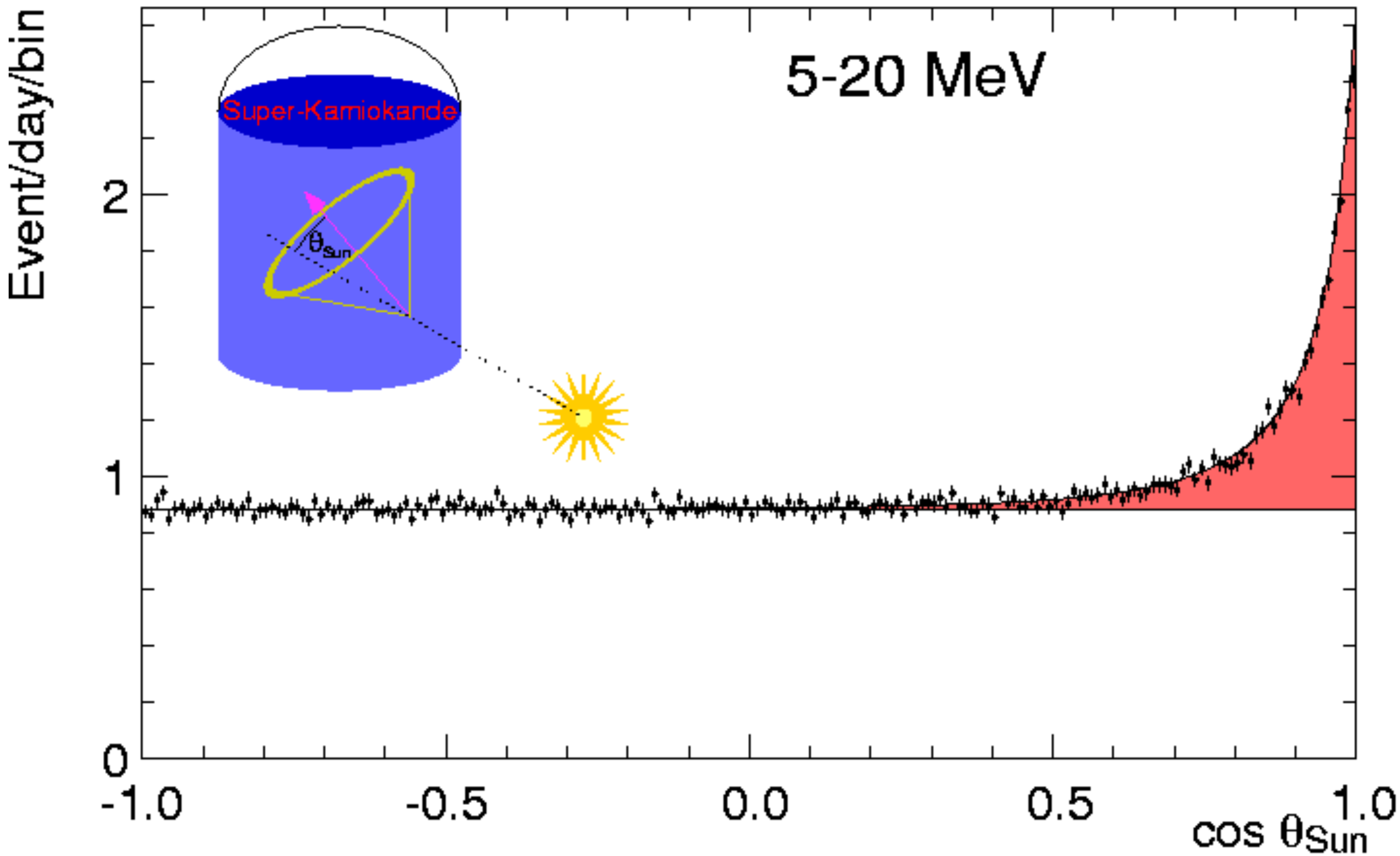


Solar neutrino experiments

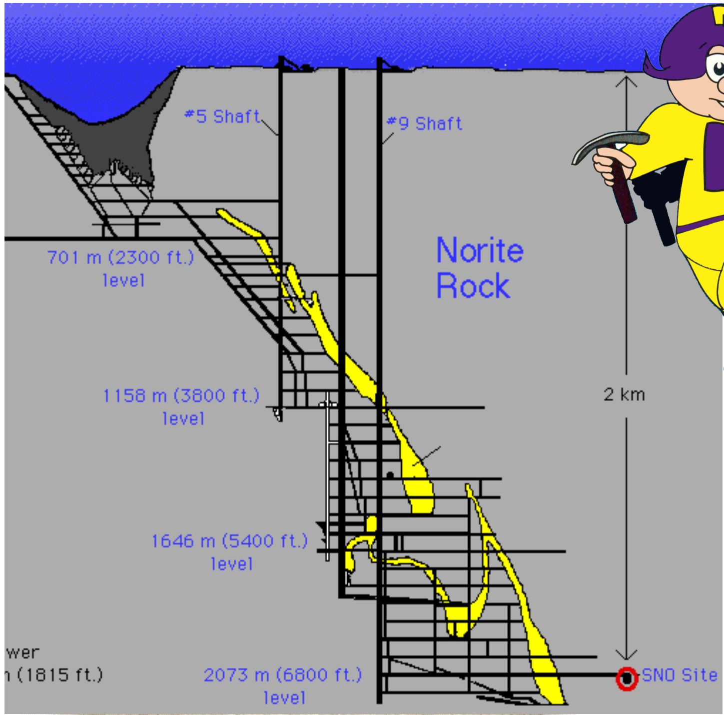


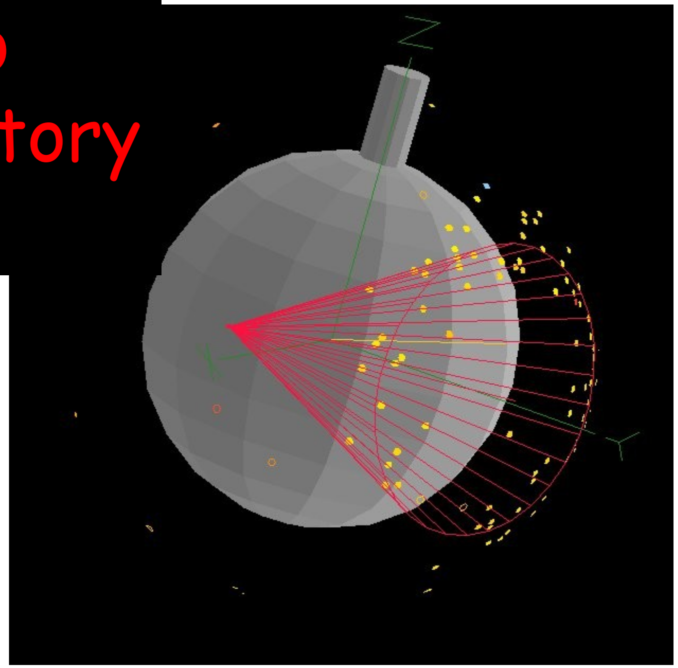


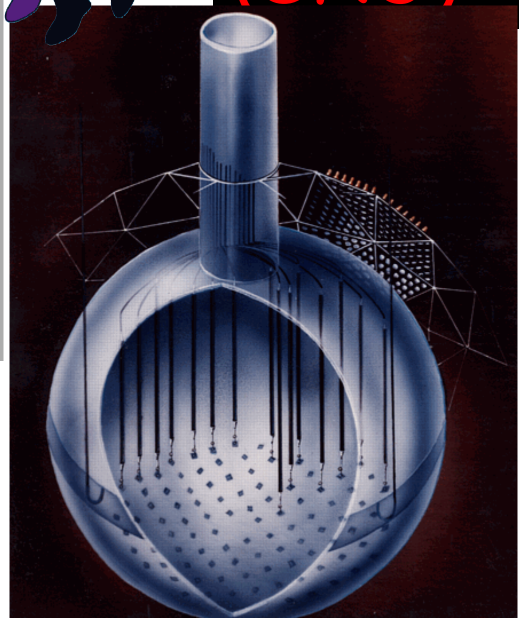
SuperKamiokande-I ^8B solar ν's

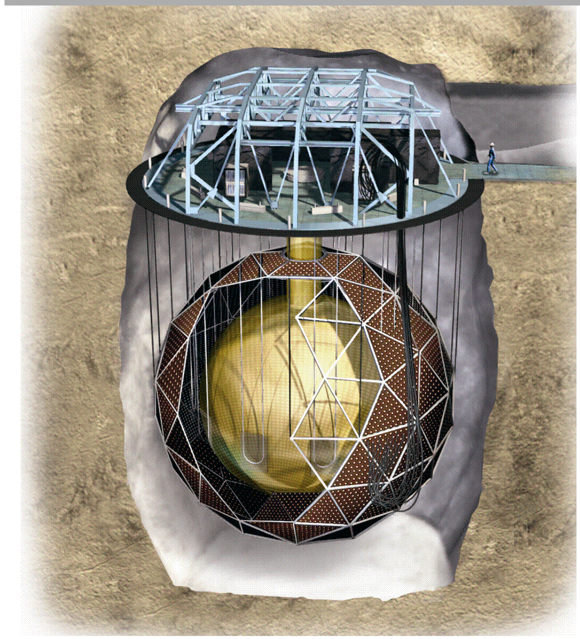


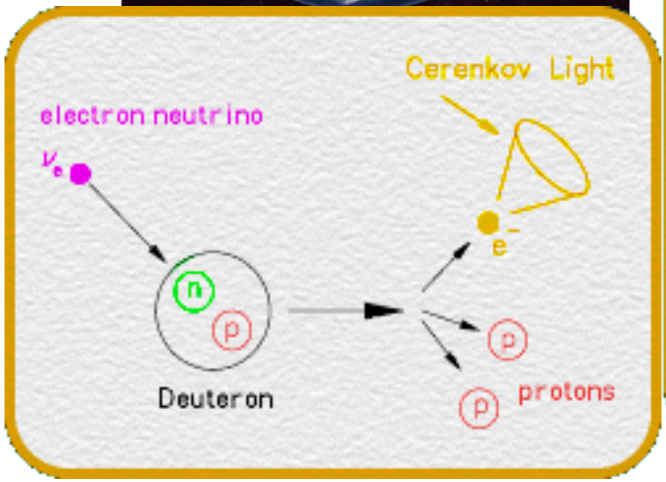
Sudbury Neutrino Observatory (SNO)

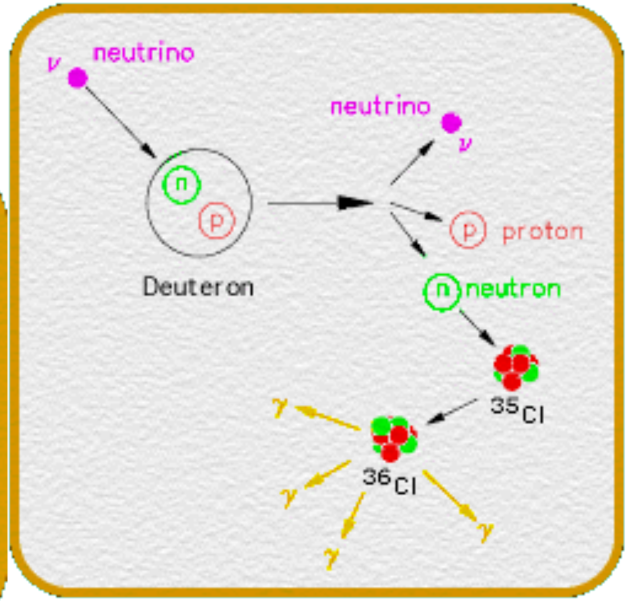






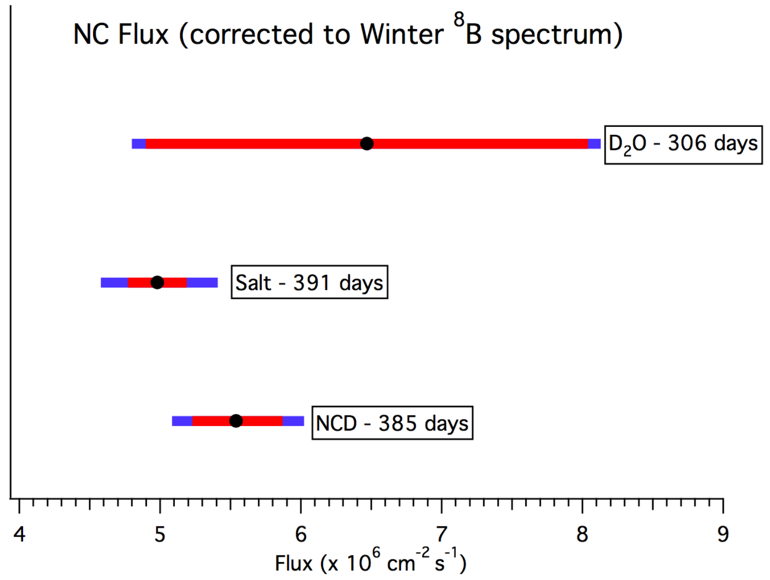


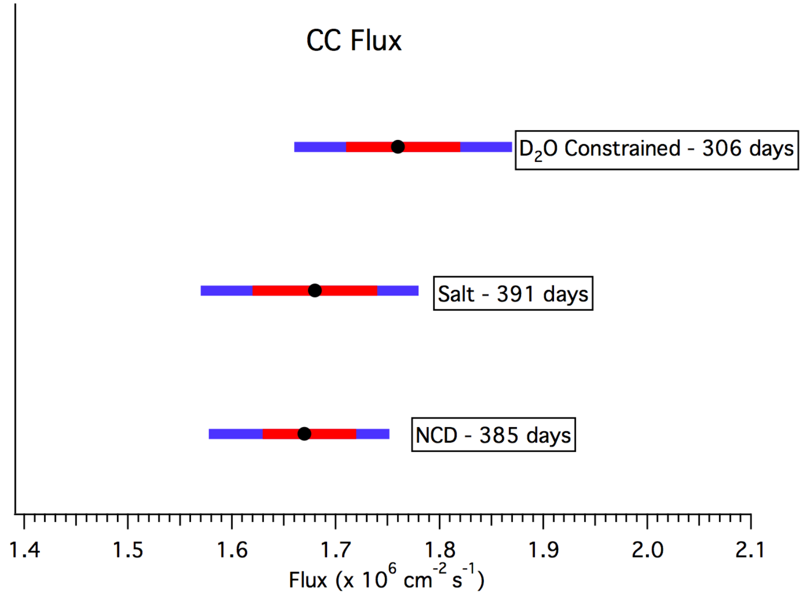


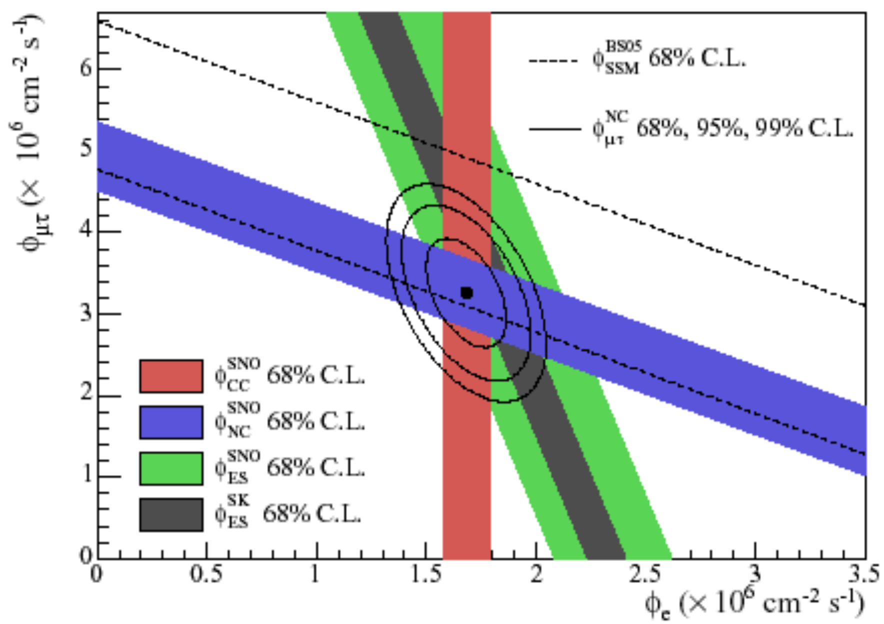


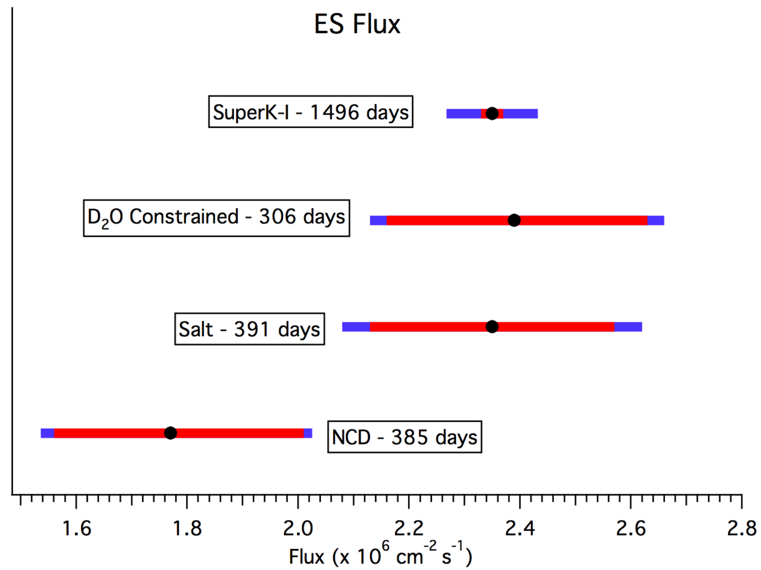
Three phases of SNO

stat — stat + syst

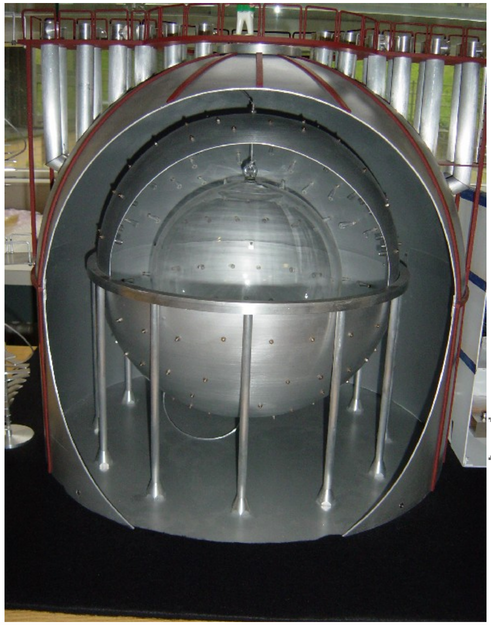


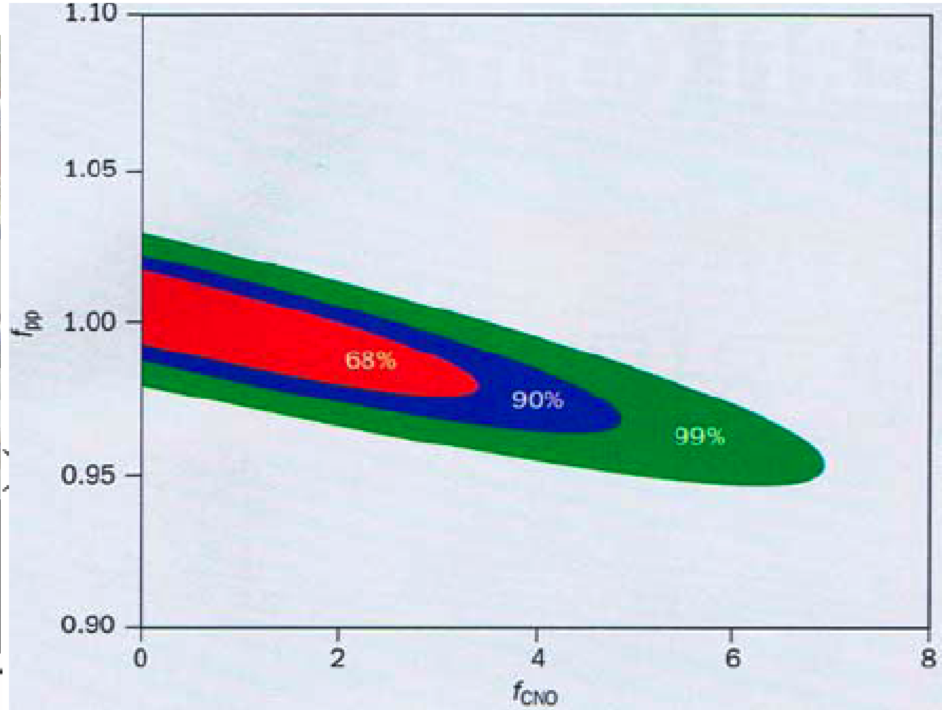


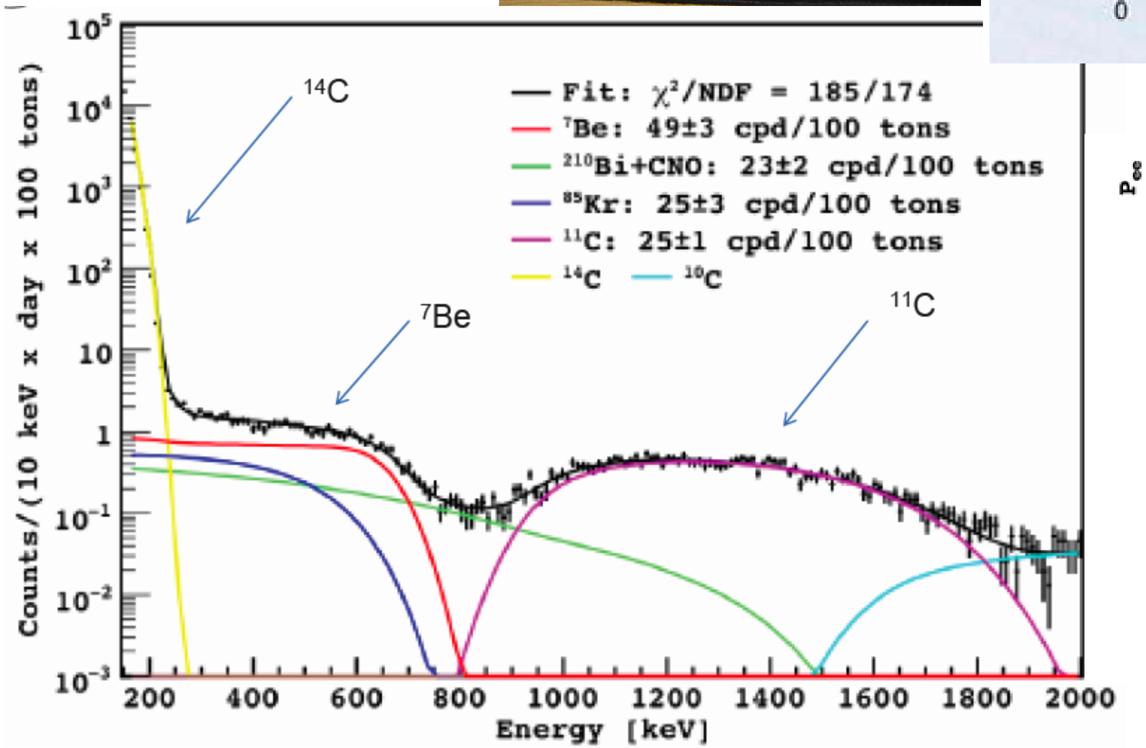


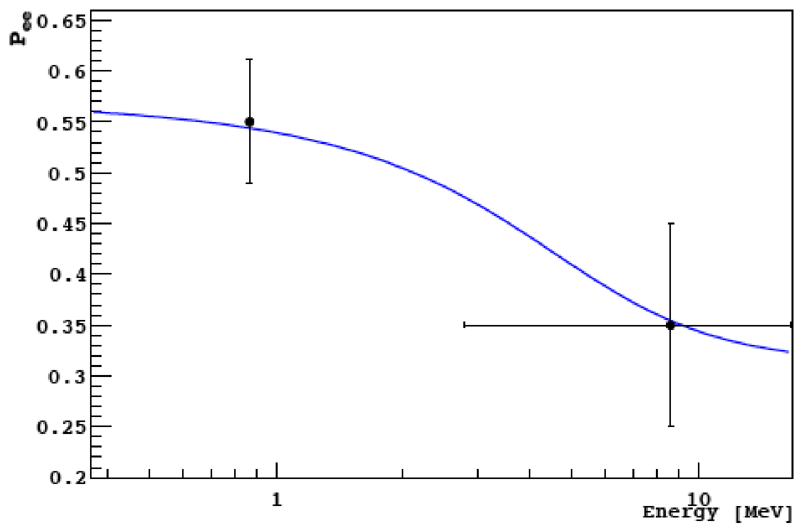


BOREXINO

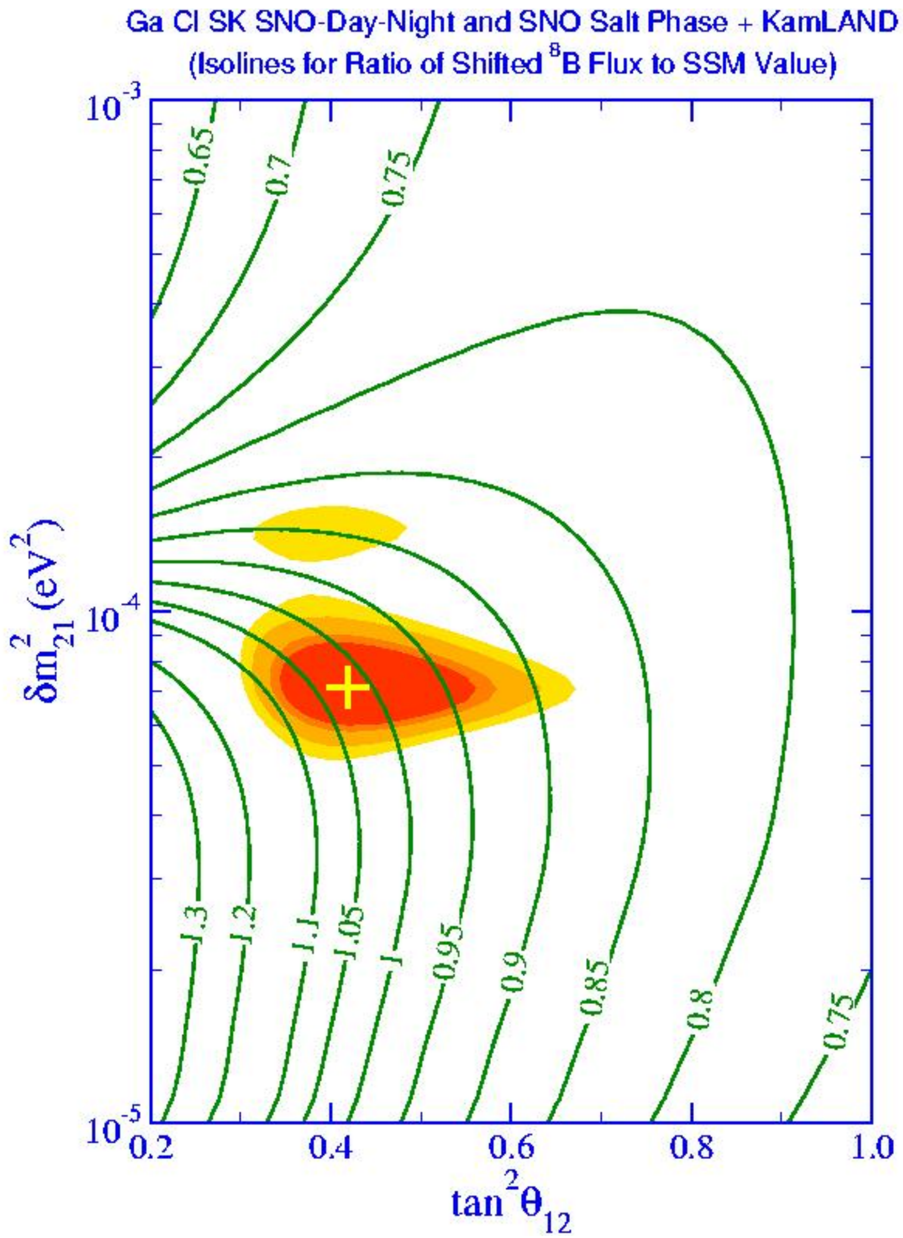


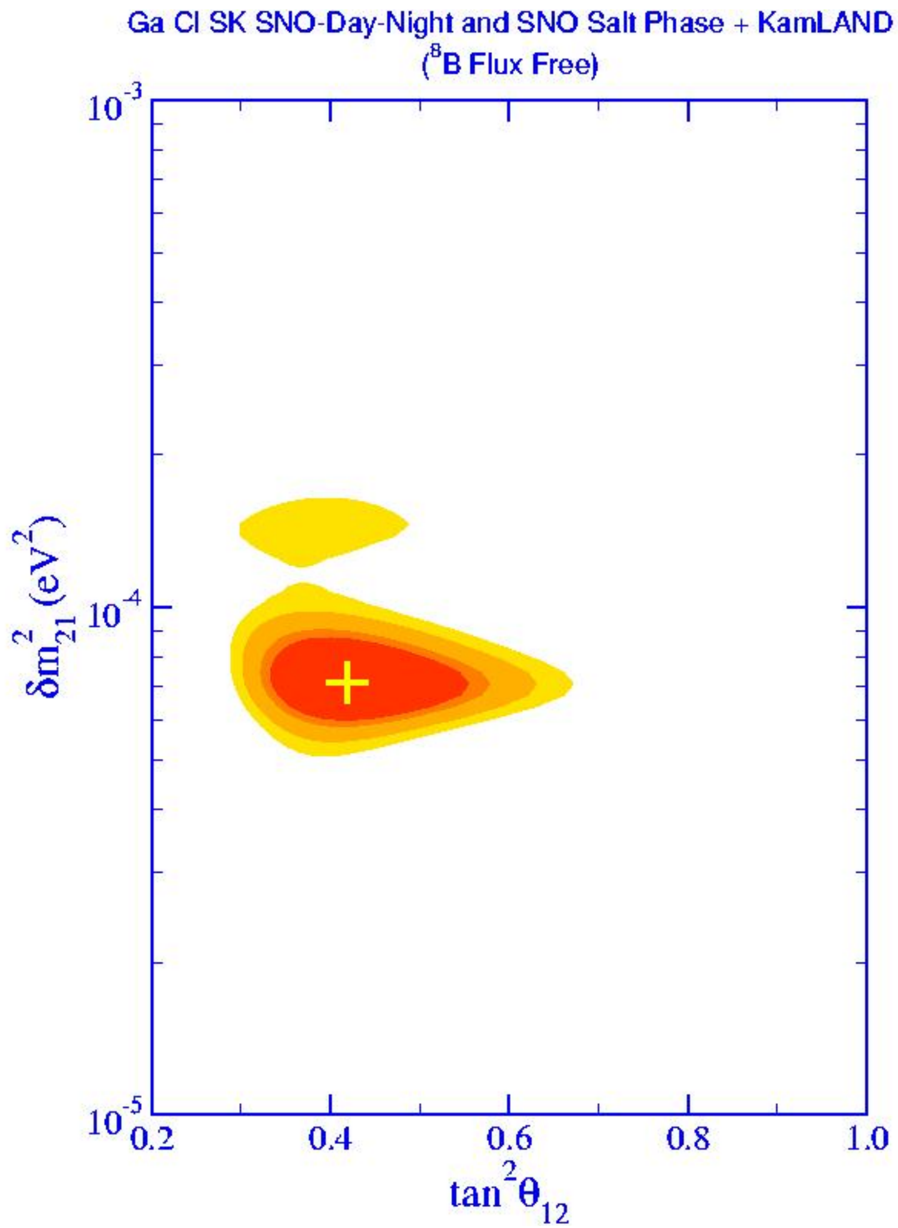

1.10
1.05
1.00
0.95
0.90
f_{pp}
68%
90%
99%
0
2
4
6
8
f_{CNO}


Fit: χ^2/NDF = 185/174
^{7}Be: 49±3 cpd/100 tons
^{210}Bi+CNO: 23±2 cpd/100 tons
^{85}Kr: 25±3 cpd/100 tons
^{11}C: 25±1 cpd/100 tons
^{14}C
^{10}C
^{14}C
^{7}Be
^{11}C
Counts/(10 keV x day x 100 tons)
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
200
400
600
800
1000
1200
1400
1600
1800
2000
Energy [keV]

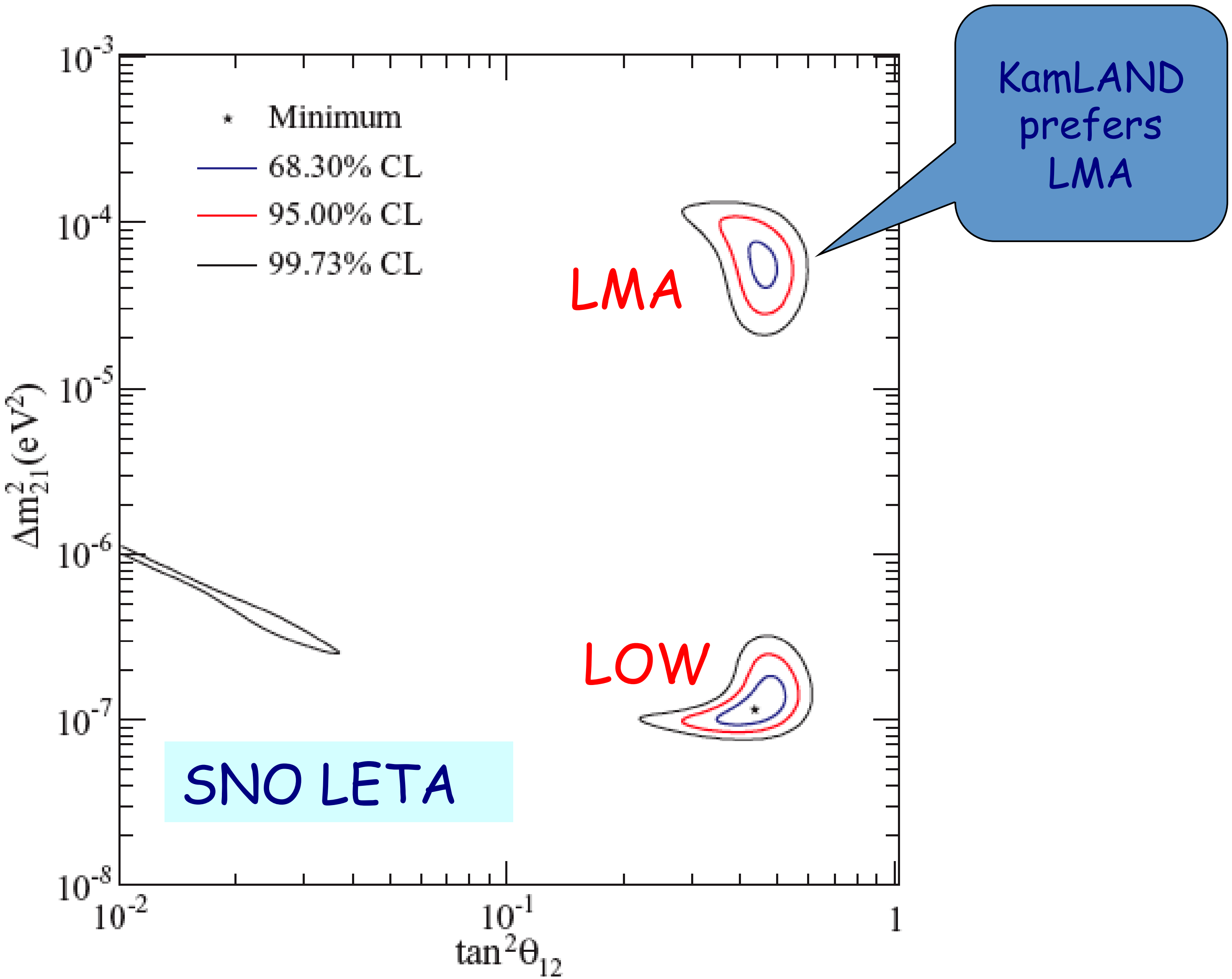

P_{ee}
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
1
10
Energy [MeV]

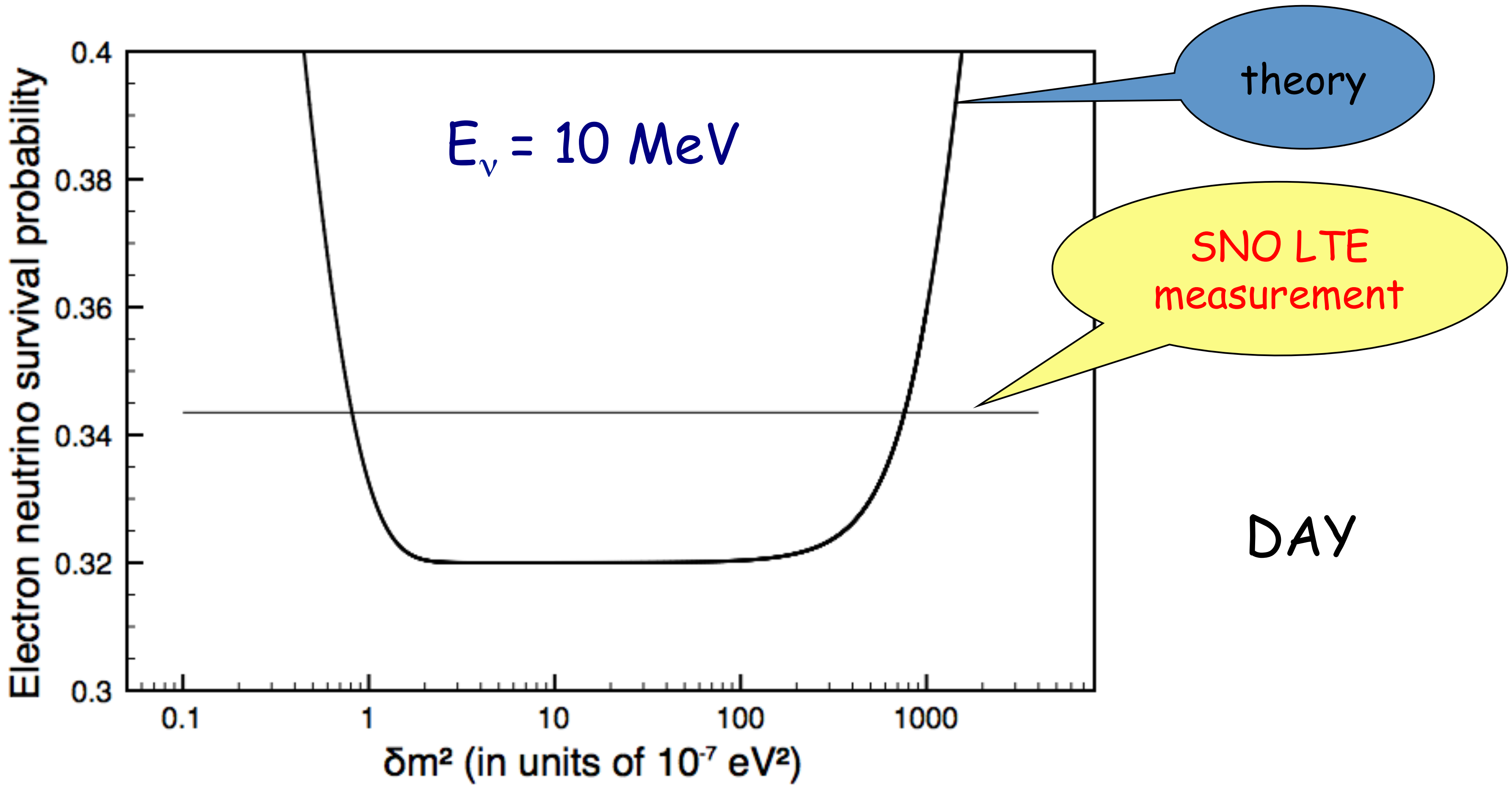
Already first SNO neutral current (salt) results could be analyzed without referring to the Standard Solar Model, A.B.B. & Yuksel, PRD 68, 113002 (2003)





Do antineutrinos mix the same way neutrinos do?





Experiments primarily sensitive to higher energy solar neutrinos cannot distinguish between LMA and LOW regions! It is desirable to pick the ***neutrino*** parameter region without KamLAND's ***antineutrinos***.

Neutrino mixing

$$\left|\nu_{flavor}\right\rangle = T\left|\nu_{mass}\right\rangle$$

$$T = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric neutrinos}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{reactor neutrinos}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar neutrinos}}$$

$$c_{ij} = \cos\theta_{ij} \qquad s_{ij} = \sin\theta_{ij}$$

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{13}\left[\cos^2\theta_{12}\sin^2(\Delta_{31}L) + \sin^2\theta_{12}\sin^2(\Delta_{32}L)\right] - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2(\Delta_{21}L)$$

$$\Delta_{ij} = \frac{\delta m_{ij}^2}{4E_\nu} = \frac{m_i^2 - m_j^2}{4E_\nu}, \qquad \Delta_{32} = \Delta_{31} - \Delta_{21}$$

The MSW Effect

In vacuum: $E^2 = \mathbf{p}^2 + m^2$

In matter:

$(E - V)^2 = (\mathbf{p} - \mathbf{A})^2 + m^2$

$\Rightarrow E^2 = \mathbf{p}^2 + m^2_{\text{eff}}$

V $\propto$ background density

$\mathbf{A} \propto \mathbf{J}_{\text{background}}$ (currents) or

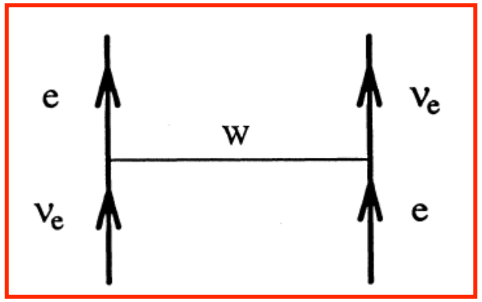
$\mathbf{A} \propto \mathbf{S}_{\text{background}}$ (spin)

In the limit of static, charge-neutral, and unpolarized background

V$\propto N_e$ and $\mathbf{A} = 0$

$\Rightarrow m^2_{\text{eff}} = m^2 + 2EV + \mathcal{O}(V^2)$

The potential is provided by the coherent forward scattering of ν_e's off the electrons in dense matter



There is a similar term with Z-exchange. But since it is the same for all neutrino flavors, it does not contribute to phase differences *unless* we invoke a sterile neutrino.

Note that matter effects induce an effective CP-violation since the matter in the Earth and the stars is not CP-symmetric!

Matter effects

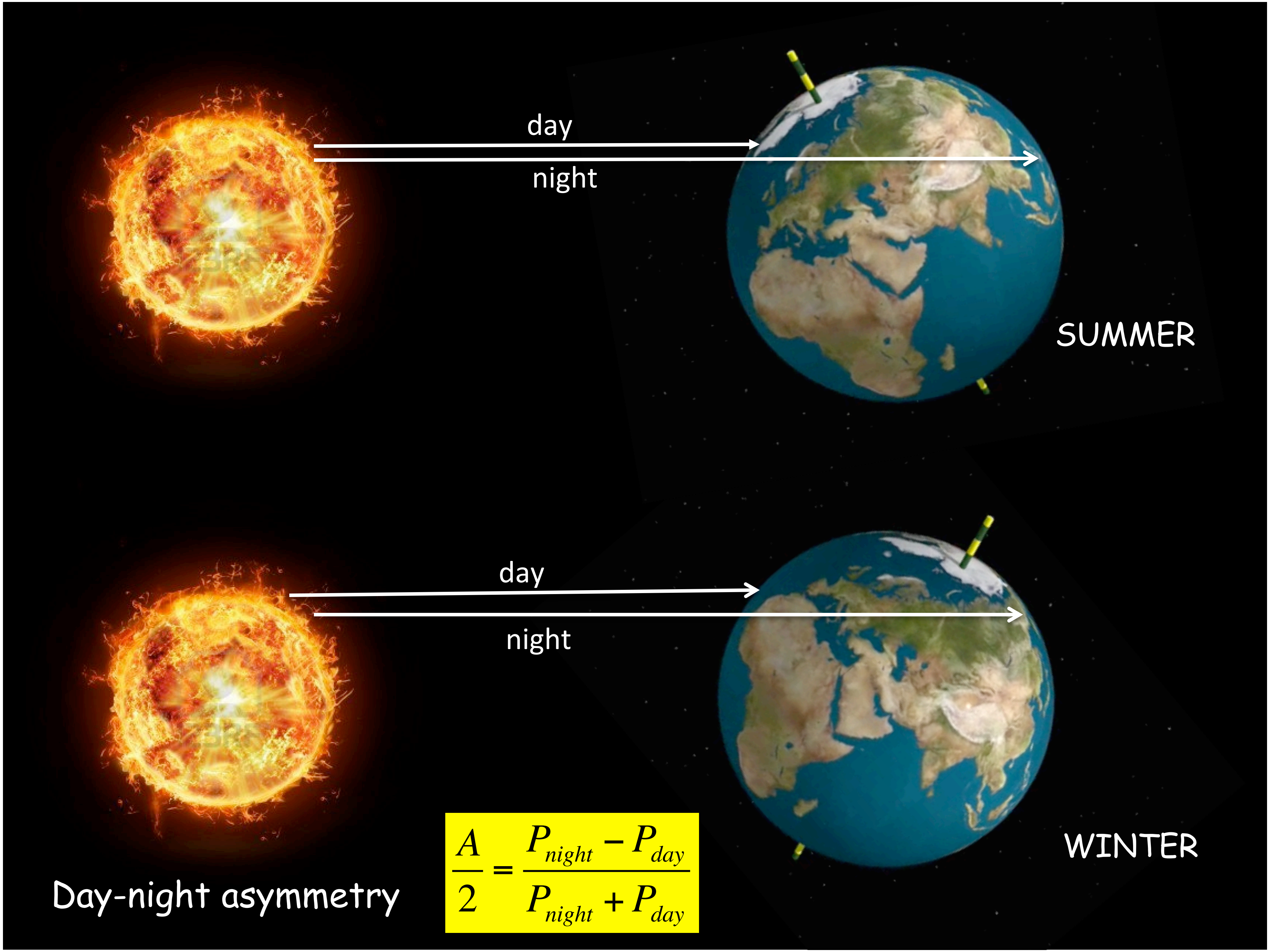
$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_e\\ \psi_\mu\\ \psi_\tau\end{pmatrix}=\left[T\begin{pmatrix}E_1&0&0\\0&E_2&0\\0&0&E_3\end{pmatrix}T^\dagger+\begin{pmatrix}V_c+V_n&0&0\\0&V_n&0\\0&0&V_n\end{pmatrix}\right]\begin{pmatrix}\psi_e\\ \psi_\mu\\ \psi_\tau\end{pmatrix}$$

$$V_c=\sqrt{2}G_F N_e \qquad V_n=-\frac{1}{\sqrt{2}}G_F N_n$$

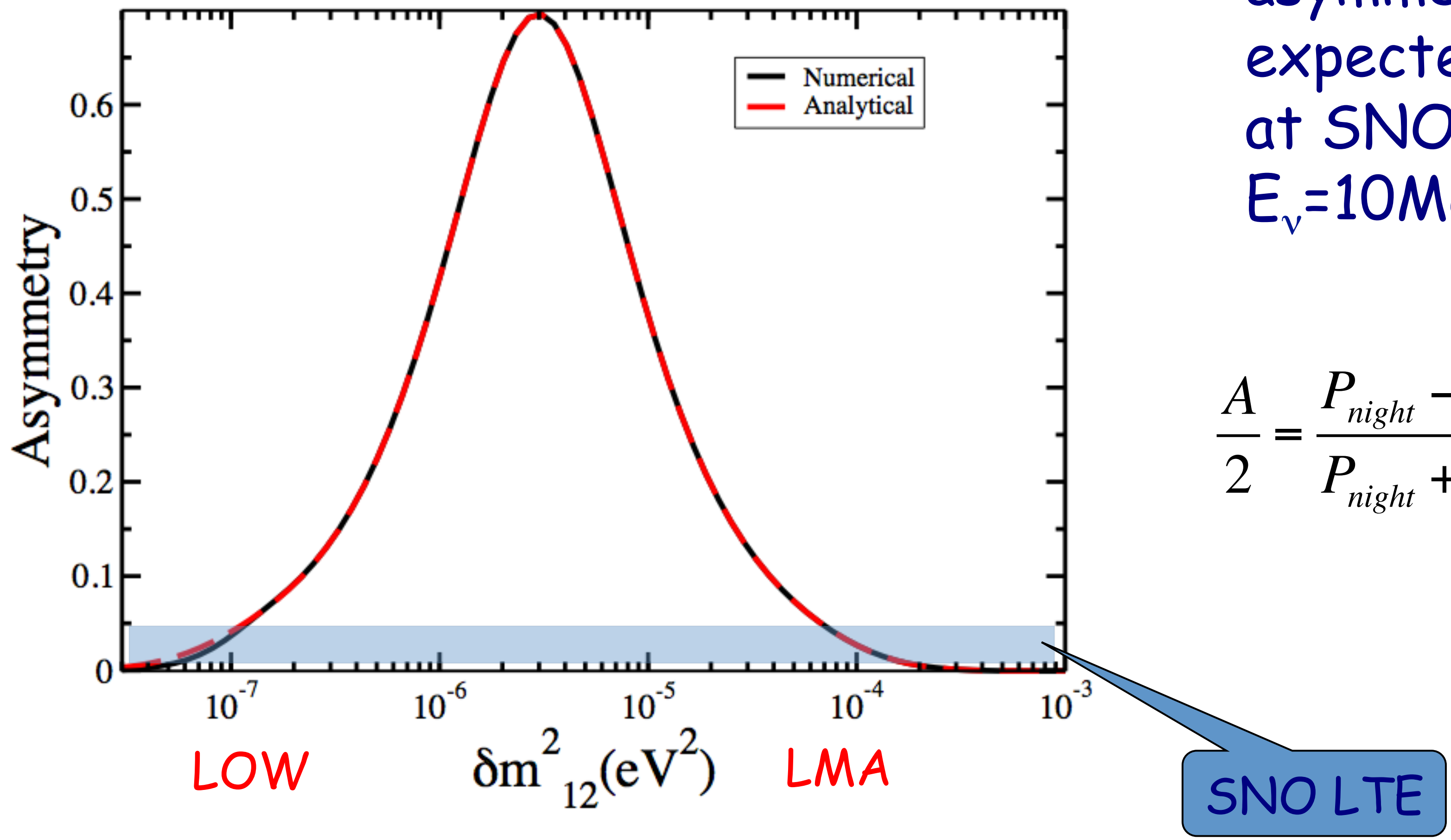
Two-flavor limit

$$i\frac{\partial}{\partial t}\begin{pmatrix}|\nu_e\rangle\\ |\nu\mu\rangle\end{pmatrix}=\begin{pmatrix}\varphi & \frac{\delta m^2}{4E}\sin 2\theta\\ \frac{\delta m^2}{4E}\sin 2\theta & -\varphi\end{pmatrix}\begin{pmatrix}|\nu_e\rangle\\ |\nu\mu\rangle\end{pmatrix}$$

$$\varphi=-\frac{\delta m^2}{4E}\cos 2\theta+\frac{1}{\sqrt{2}}G_F N_e$$

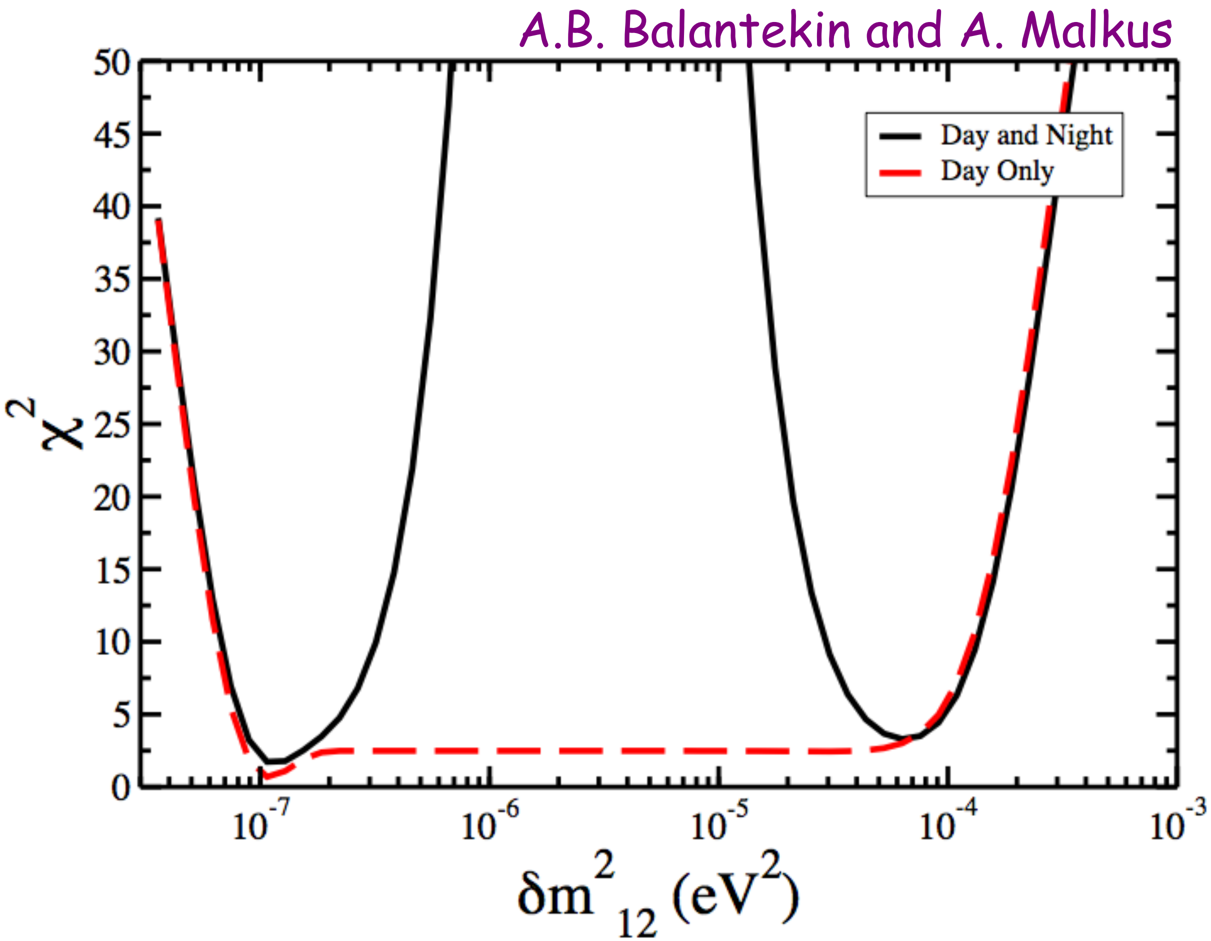

day
night
SUMMER
day
night
WINTER
Day-night asymmetry
$\frac{A}{2} = \frac{P_{night} - P_{day}}{P_{night} + P_{day}}$

Day-night asymmetry expected at SNO for E_ν=10MeV



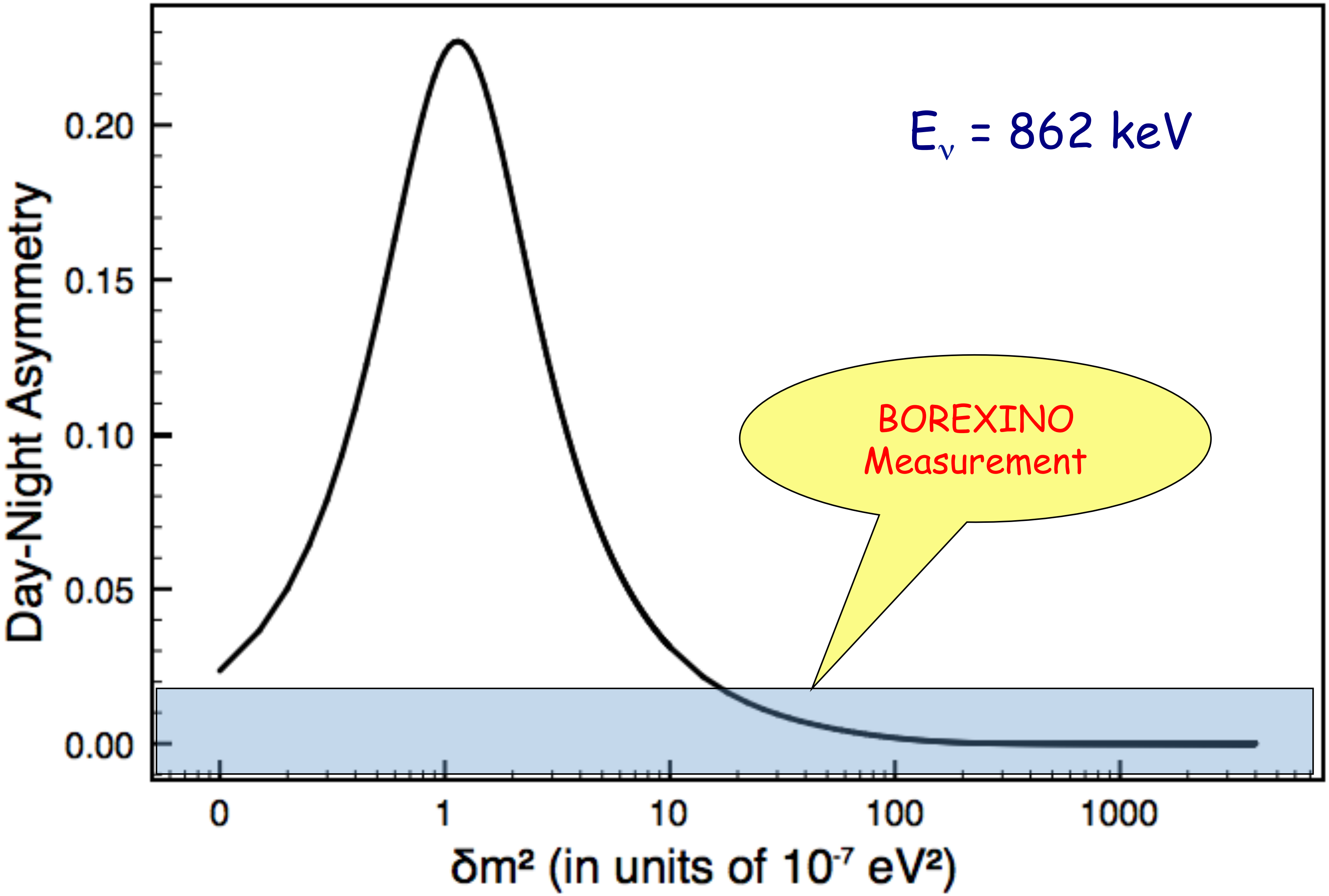
$$\frac{A}{2} = \frac{P_{night} - P_{day}}{P_{night} + P_{day}}$$

Experiments primarily sensitive to higher energy solar neutrinos cannot distinguish between LMA and LOW regions! It is desirable to pick the ***neutrino*** parameter region without KamLAND's ***antineutrinos***.

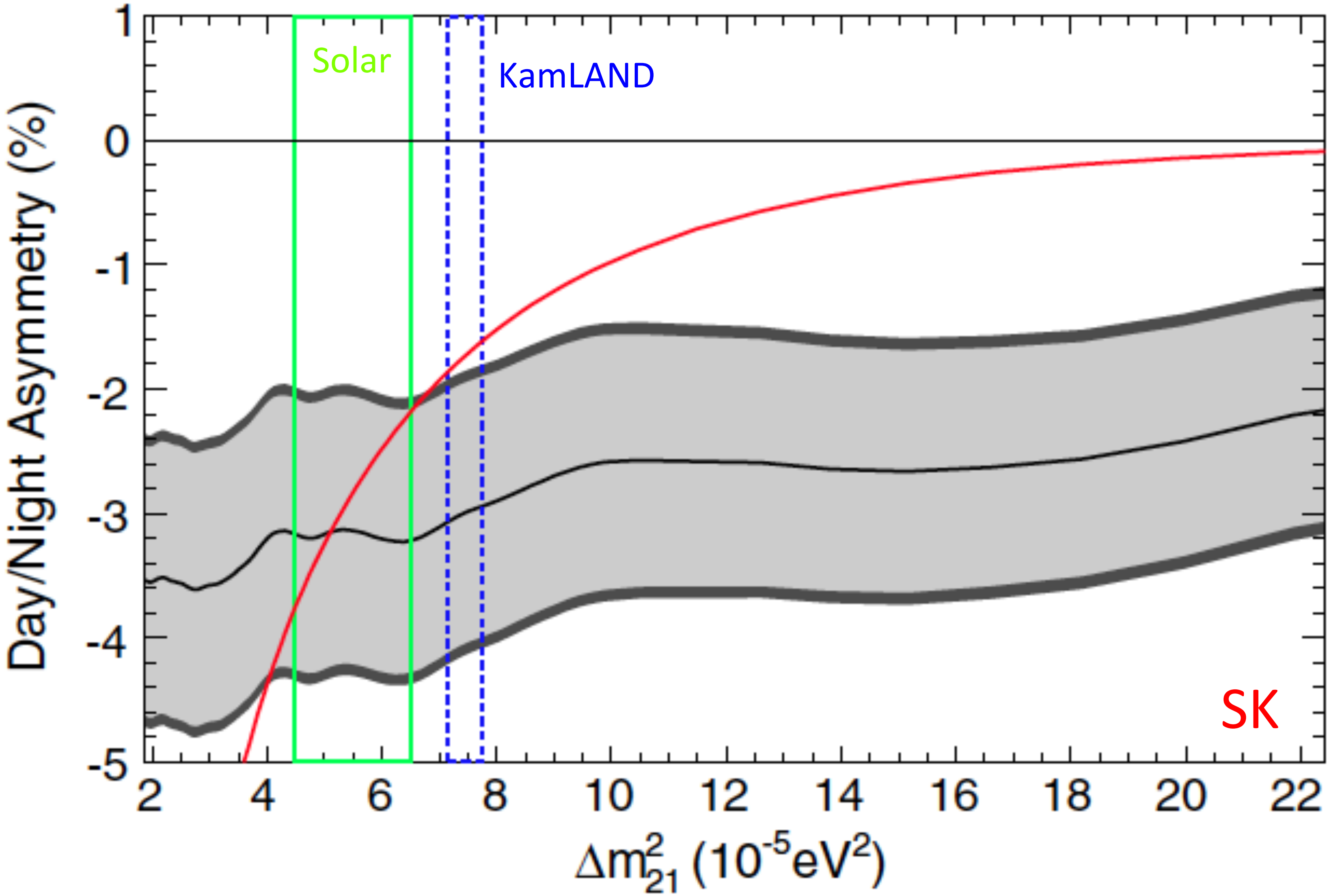


Fit to the SNO LTE data

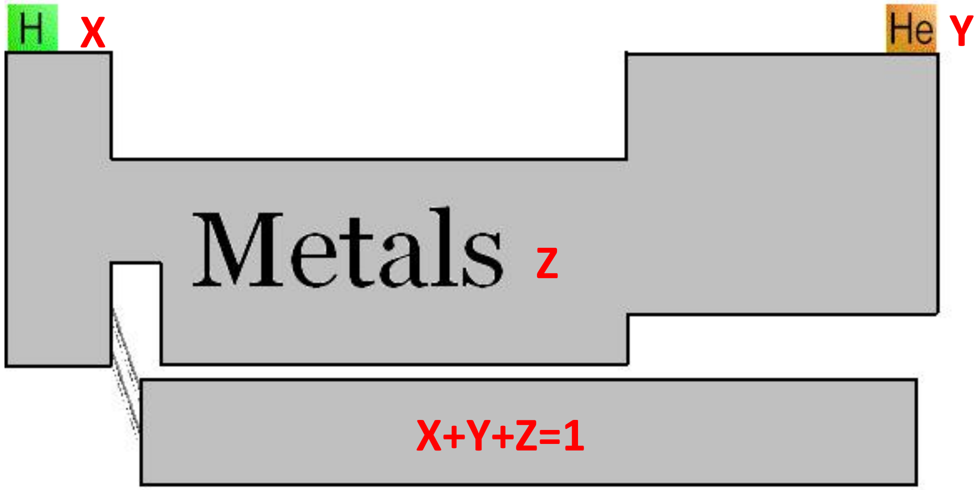
Experiments primarily sensitive to higher energy solar neutrinos cannot distinguish between LMA and LOW regions! It is desirable to pick the ***neutrino*** parameter region without KamLAND's ***antineutrinos***.



E_ν = 862 keV
BOREXINO Measurement
Day-Night Asymmetry
0.20
0.15
0.10
0.05
0.00
0
1
10
100
1000
δm^2 (in units of 10^{-7} eV^2)



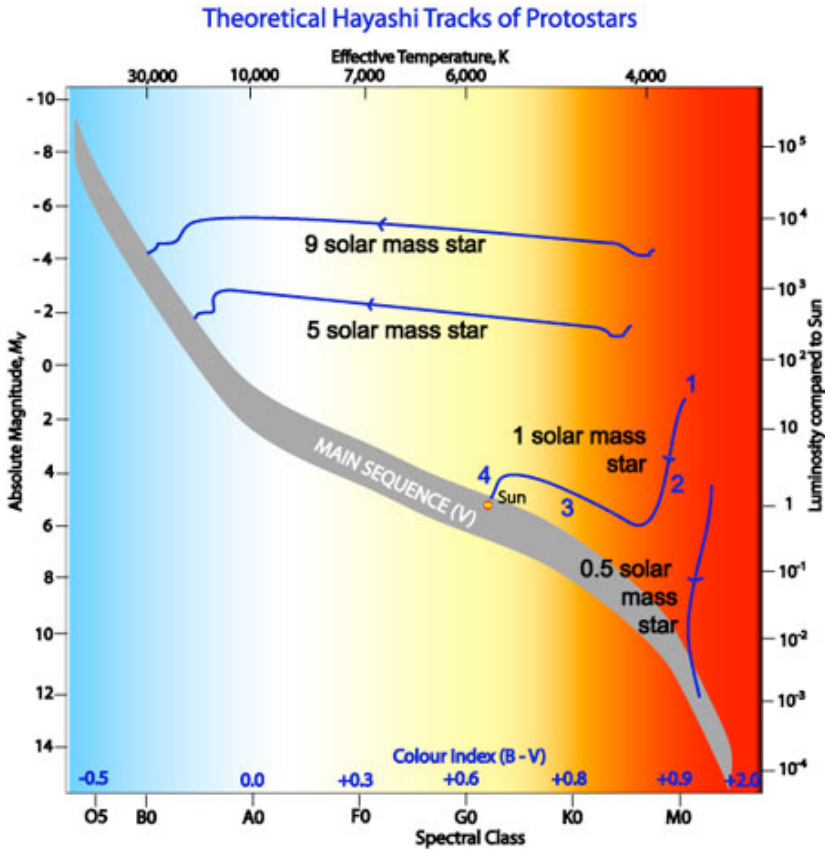
PRL **112,** 091805 (2014)



SSM assumption: The proto-Sun follows the convective Hayashi track ➔ zero-age Sun is homogeneous, i.e $Z_{initial} = Z_{surface\_today}$

Initial parameters: $Y_{initial}$, $Z_{initial}$, solar mixing length

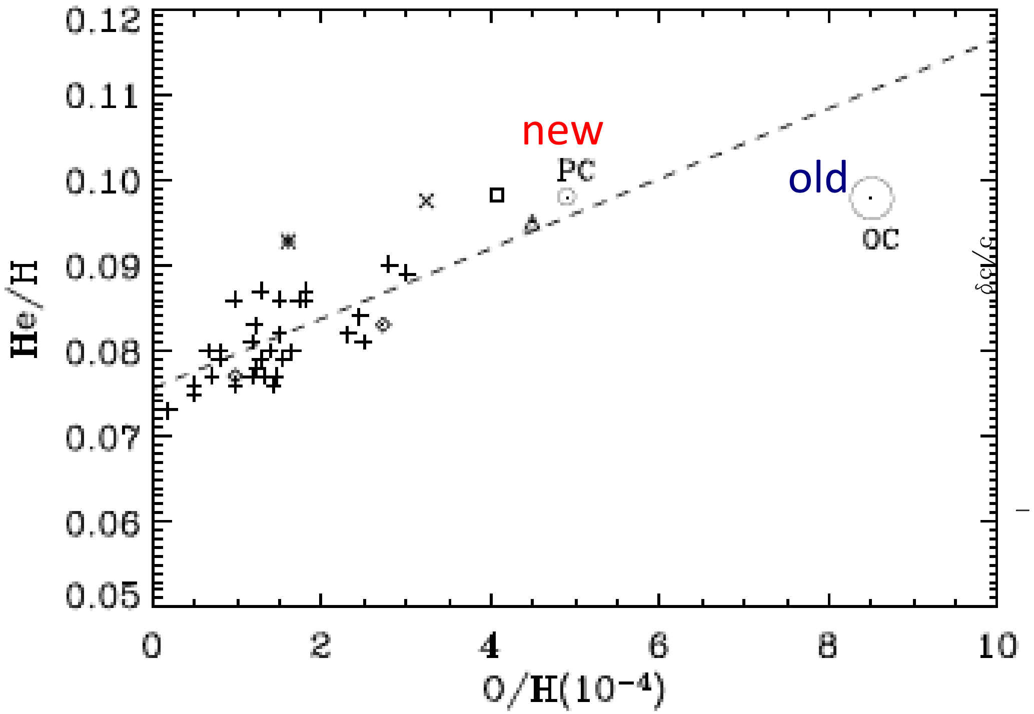
Evolve forward to today to reproduce present $R_\odot$, $L_\odot$, and $Y_{surface}$

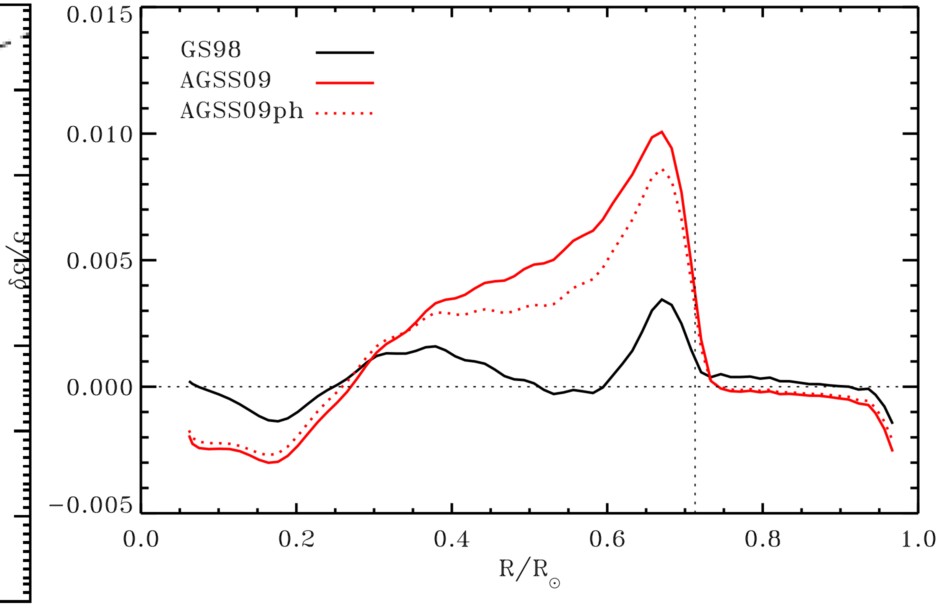


$Z_{surface\_today}$ is deduced from photospheric absorption lines, which were recently evaluated using 3D methods. $Z_{surface\_today}$ obtained using improved methods does not match $Z_{initial}$ of the SSM!

This fixes some old puzzles

But creates new ones!





Sun is no longer an "odd" star enriched in heavy elements!

Old ^{8}B neutrino flux = 4x10^{6} cm^{-2}s^{-1}
New ^{8}B neutrino flux = 5.31x10^{6} cm^{-2}s^{-1}

There is mismatch between the surface and the interior of the Sun!

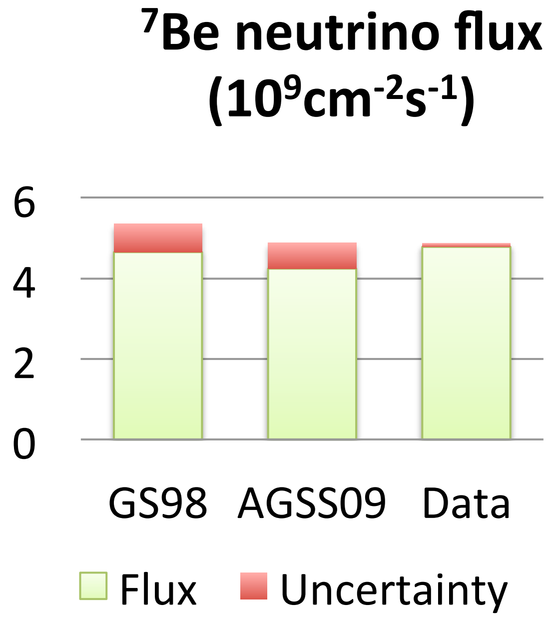
CNO Neutrinos are still not measured!

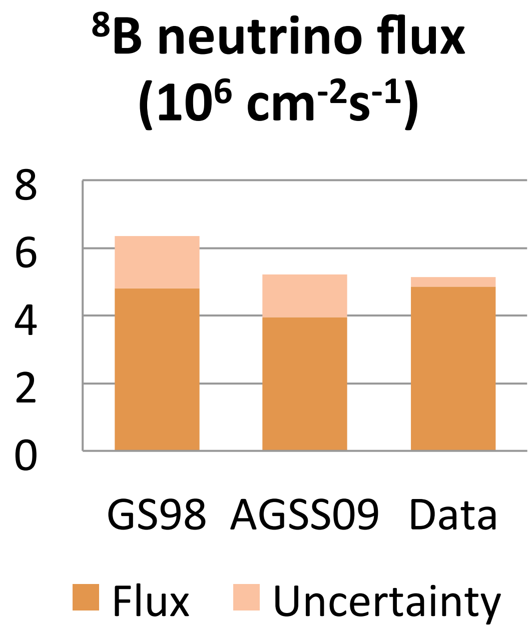
New Solar abundances:

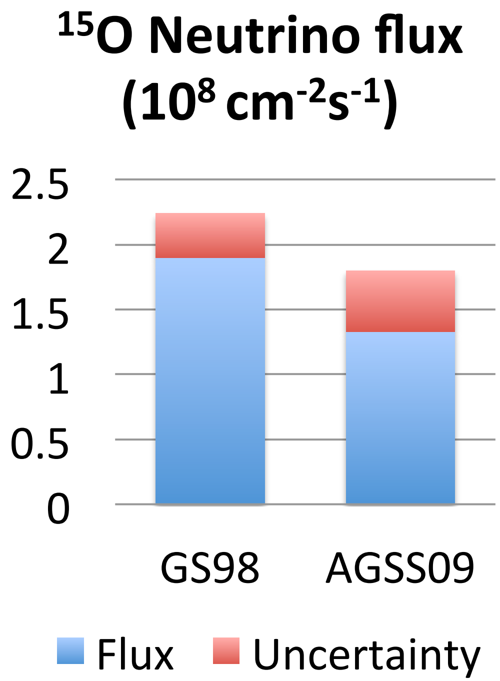
- Asplund *et al.* (AGSS09), $(Z/X)_\odot=0.0178$
- Grevesse and Sauvel (GS98), $(Z/X)_\odot=0.0229$

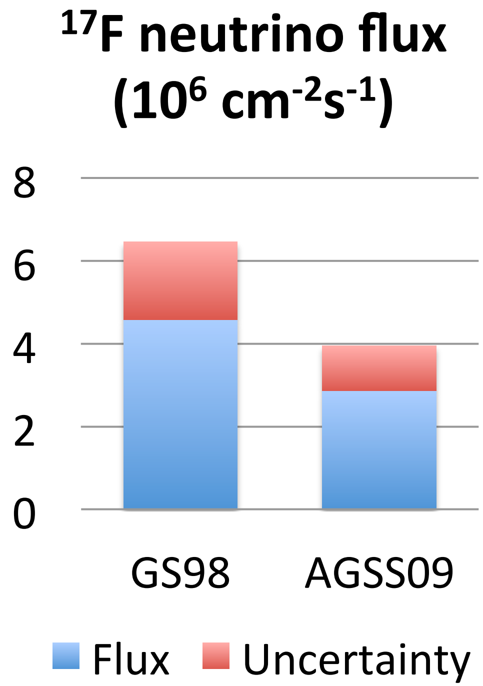
Drastically different!
Open problem in solar physics!

- New Evaluation of the nuclear reaction rates: Adelberger et al. (2011)
- New solar model calculations:Serenelli



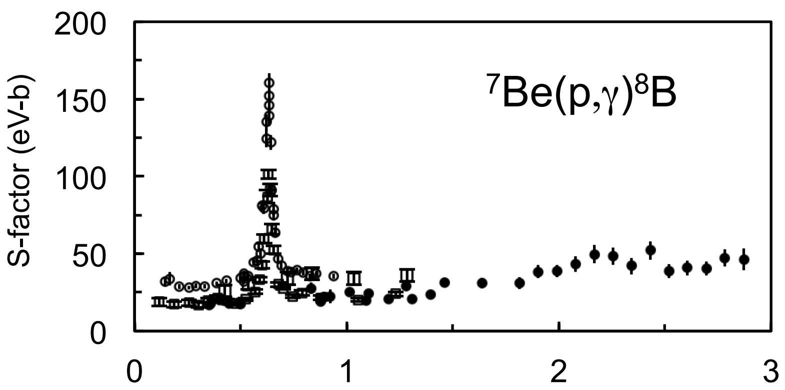


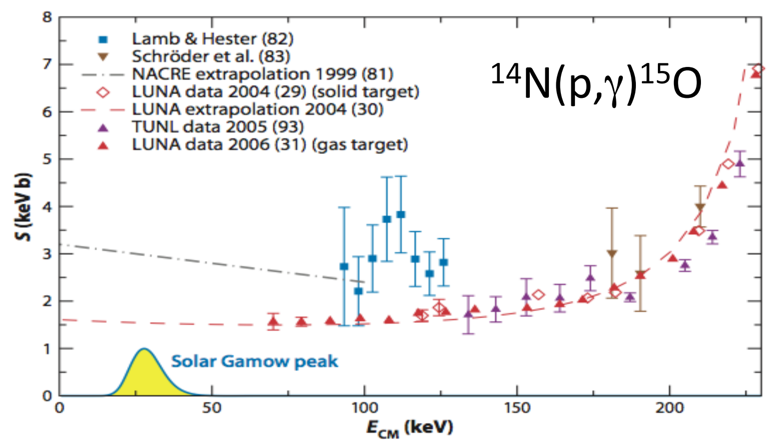




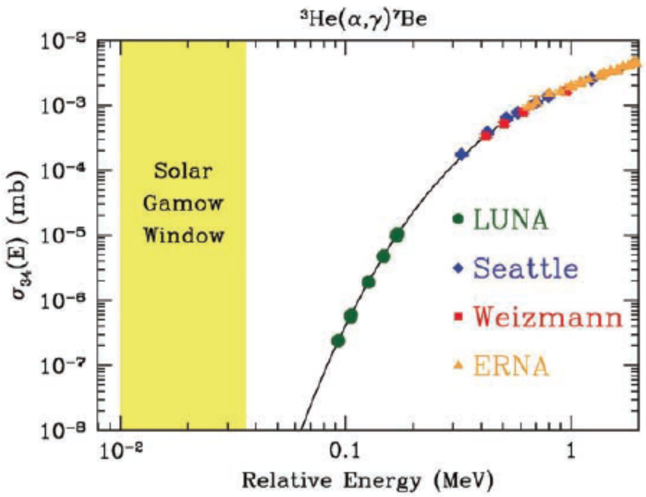
SSM Error Budget

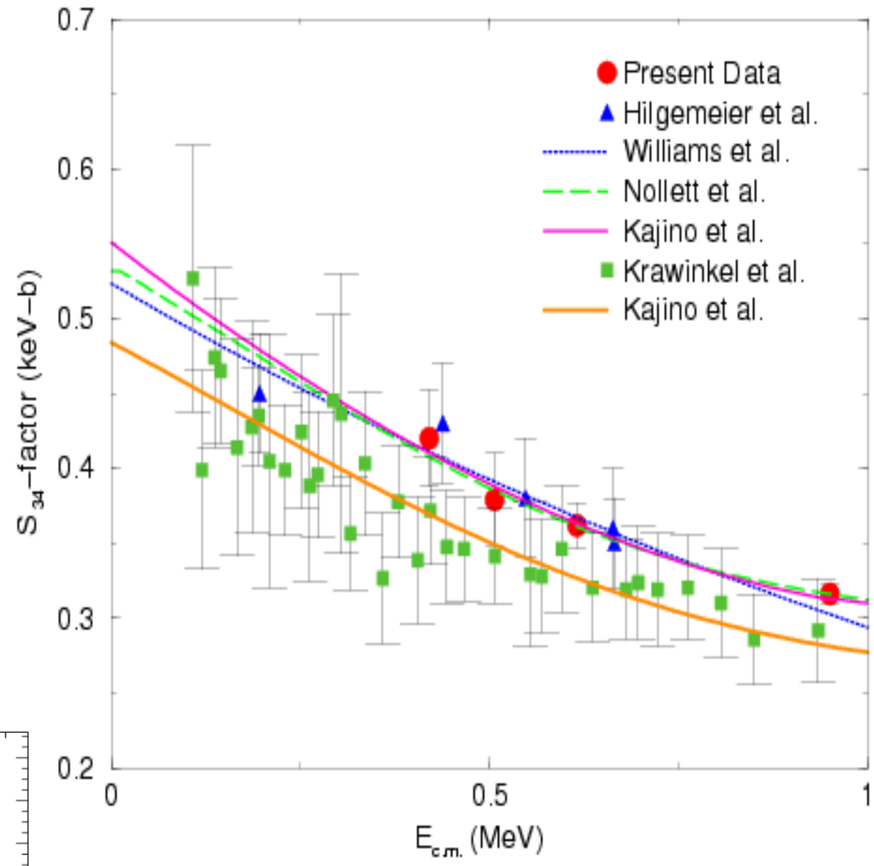
Source	Percentage Error
Diffusion coefficient of SSM	2.7%
Nuclear rates [mainly $^{7}Be(p,\gamma)^{8}B$ and $^{14}N(p,\gamma)^{15}O$]	9.9%
Neutrinos and weak interaction (mainly θ_{12})	3.2%
Other SSM input parameters	0.6%

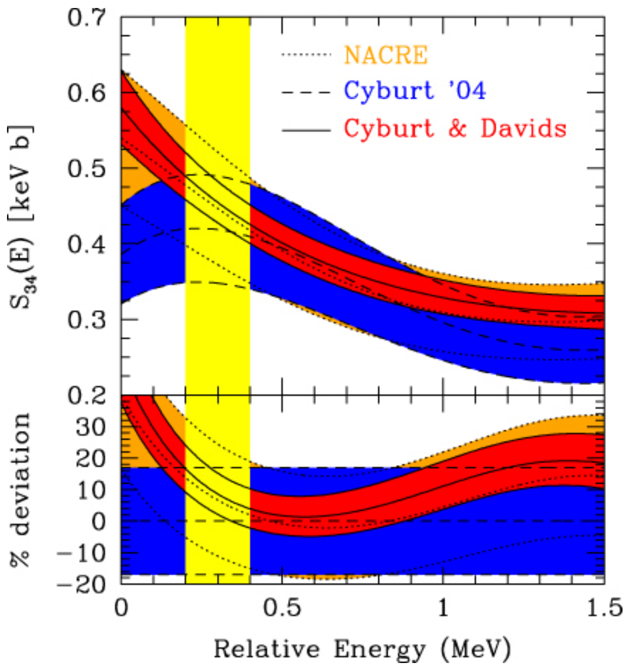


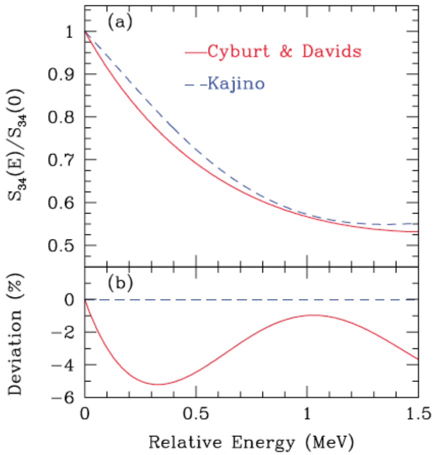


$^{3}He(\alpha,\gamma)^{7}Be$

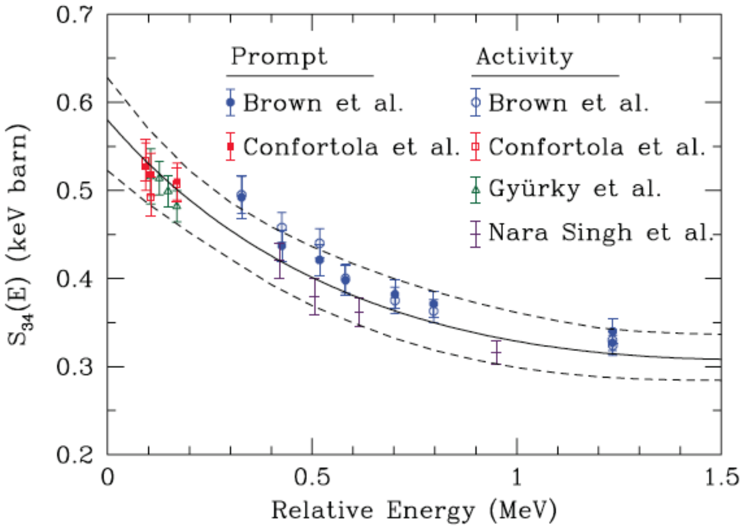




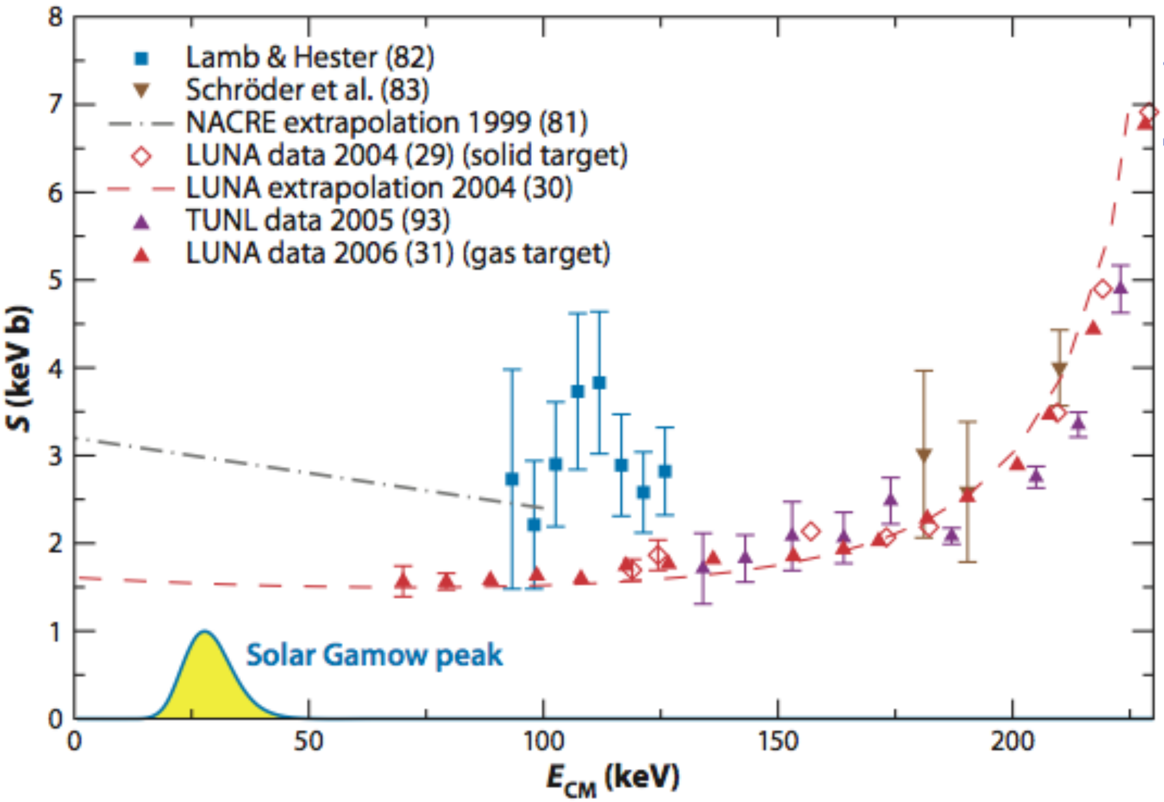


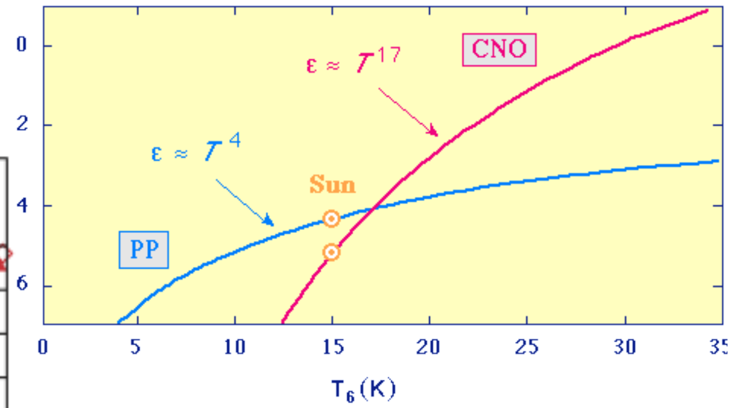


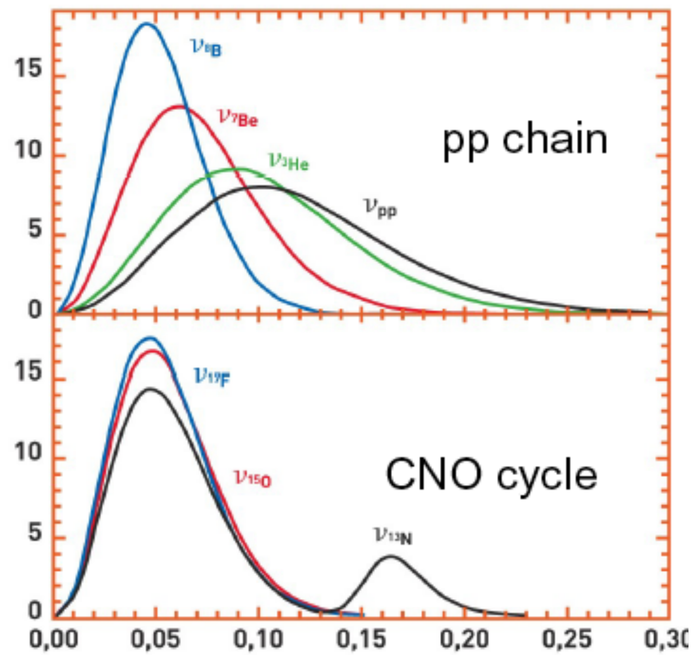
The main uncertainty for the Sun and Big-Bang nucleosynthesis



$^{14}N(p,\gamma)^{15}O$







The determining reaction for the CNO burning