Collective Neutrino Oscillations

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THE UNIVERSITY *of* NEW MEXICO

International Summer School on AstroComputing 2014
Neutrino & Nuclear Astrophysics

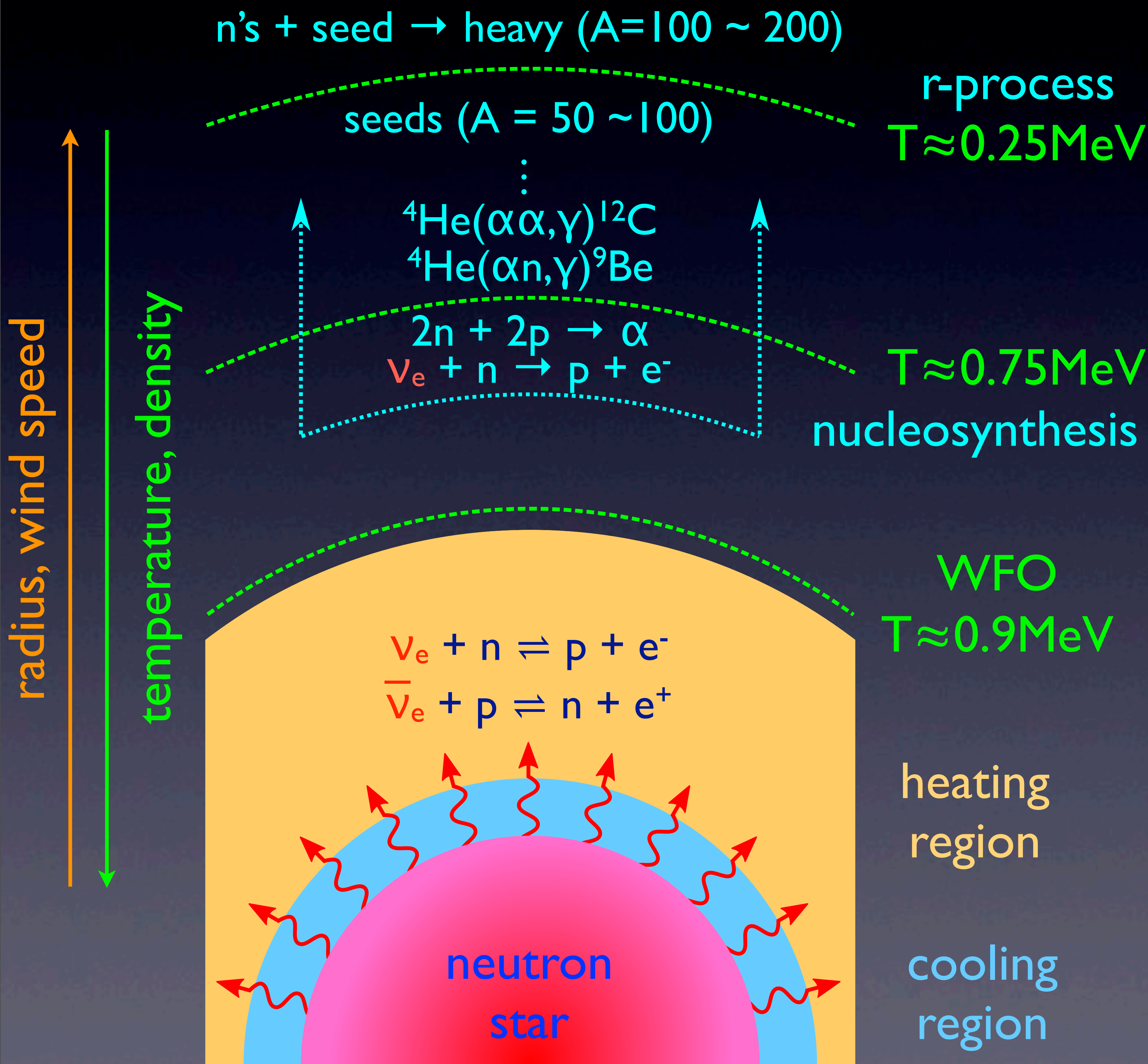
Outline

✦ Introduction & overview

✦ Understandings & insights

✦ New developments & challenges

Neutrinos in Supernovae

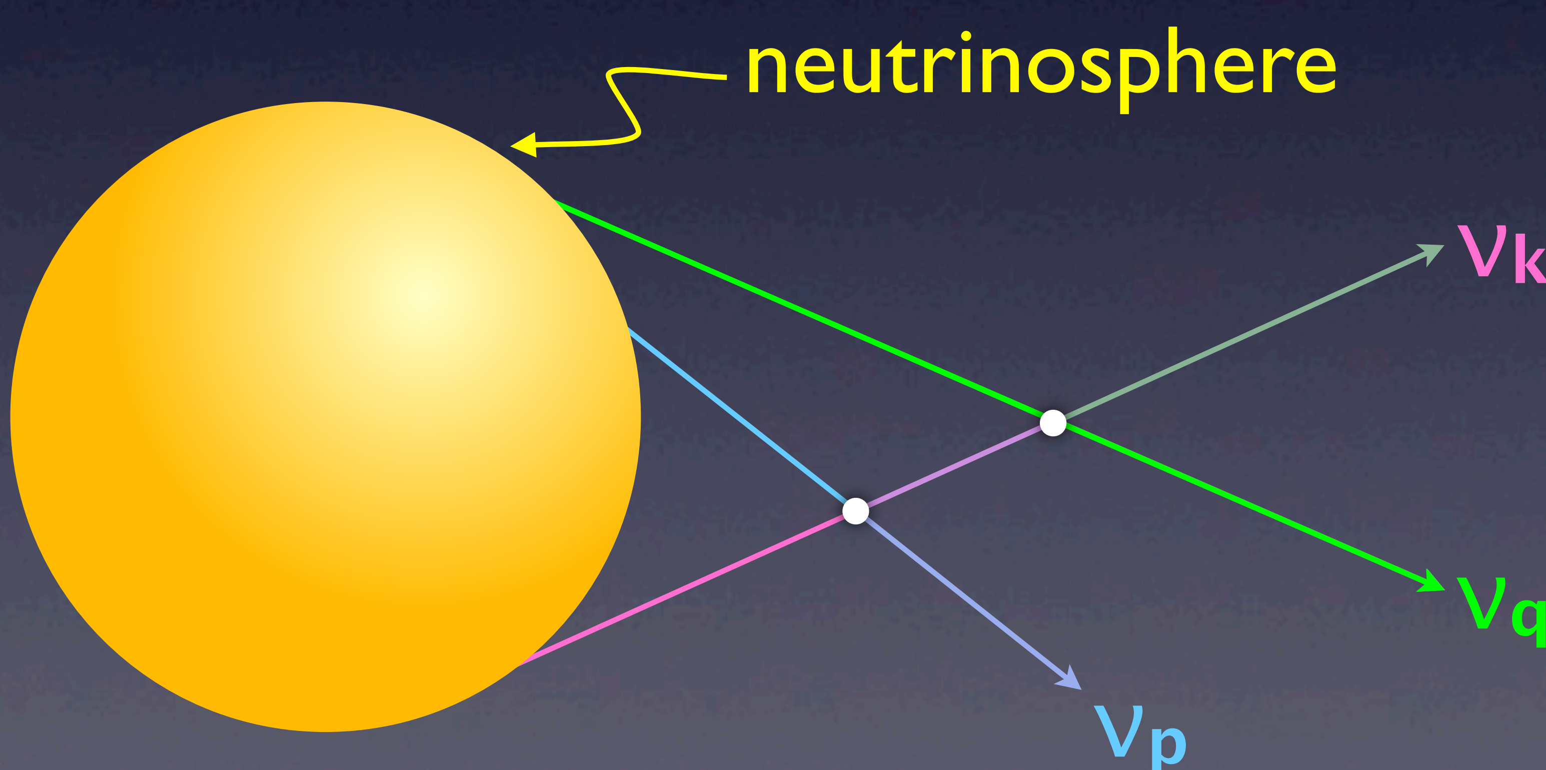


- ~10^{53} ergs, 10^{58} neutrinos in ~10 seconds
- All neutrino species, 10~30 MeV
- Dominate energetics
- Influence nucleosynthesis
- Probe into SNe

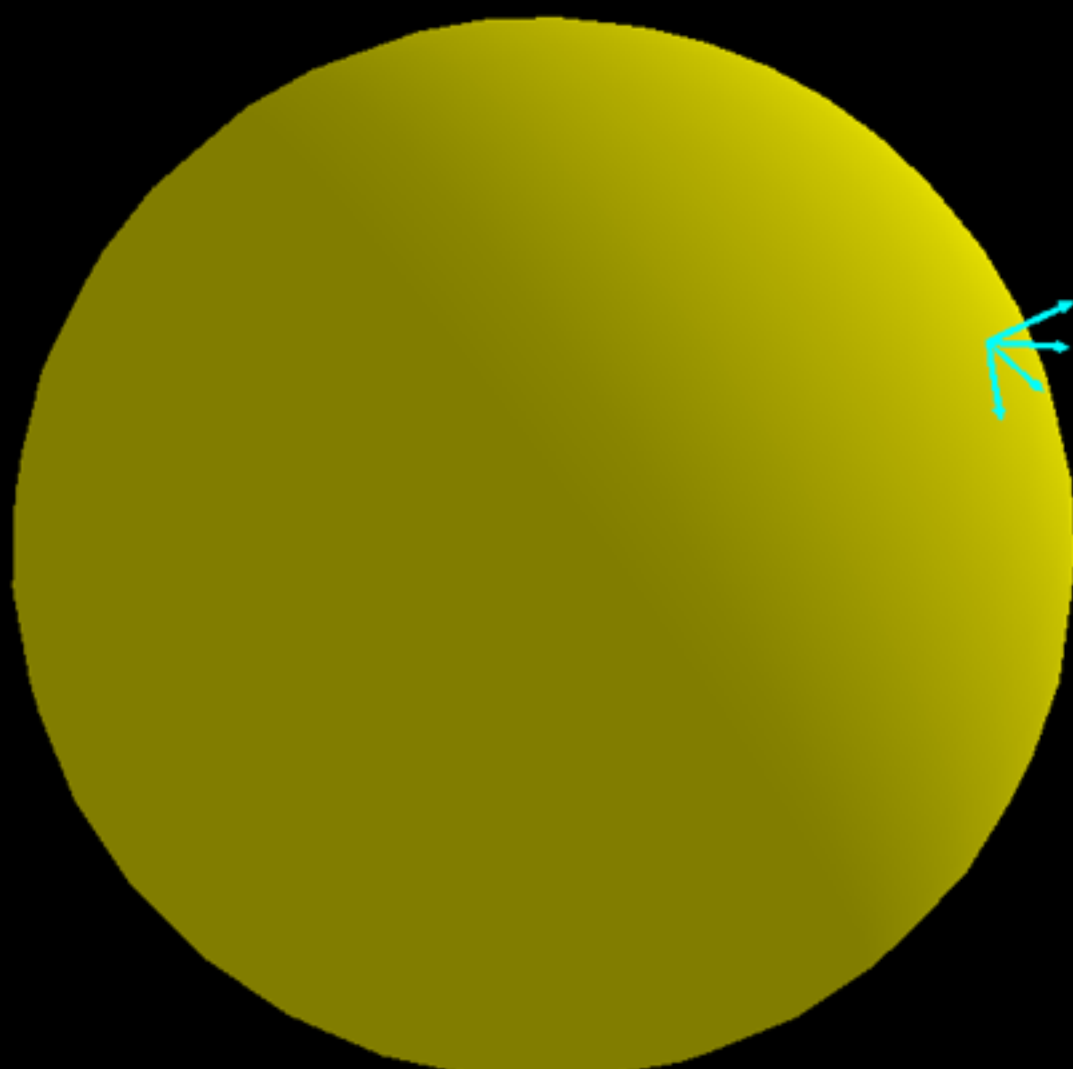
ν oscillations in SN

$$i\frac{d}{d\lambda}|\psi_{\nu,\mathbf{p}}\rangle = \hat{H}|\psi_{\nu,\mathbf{p}}\rangle$$

$$\mathrm{H} = \frac{\mathrm{M}^2}{2E} + \sqrt{2}G_\mathrm{F}\,\mathrm{diag}[n_e, 0, 0] + \mathrm{H}_{\nu\nu}$$



(1+2)D
Multi-Angle/Bulb Model



propagation direction

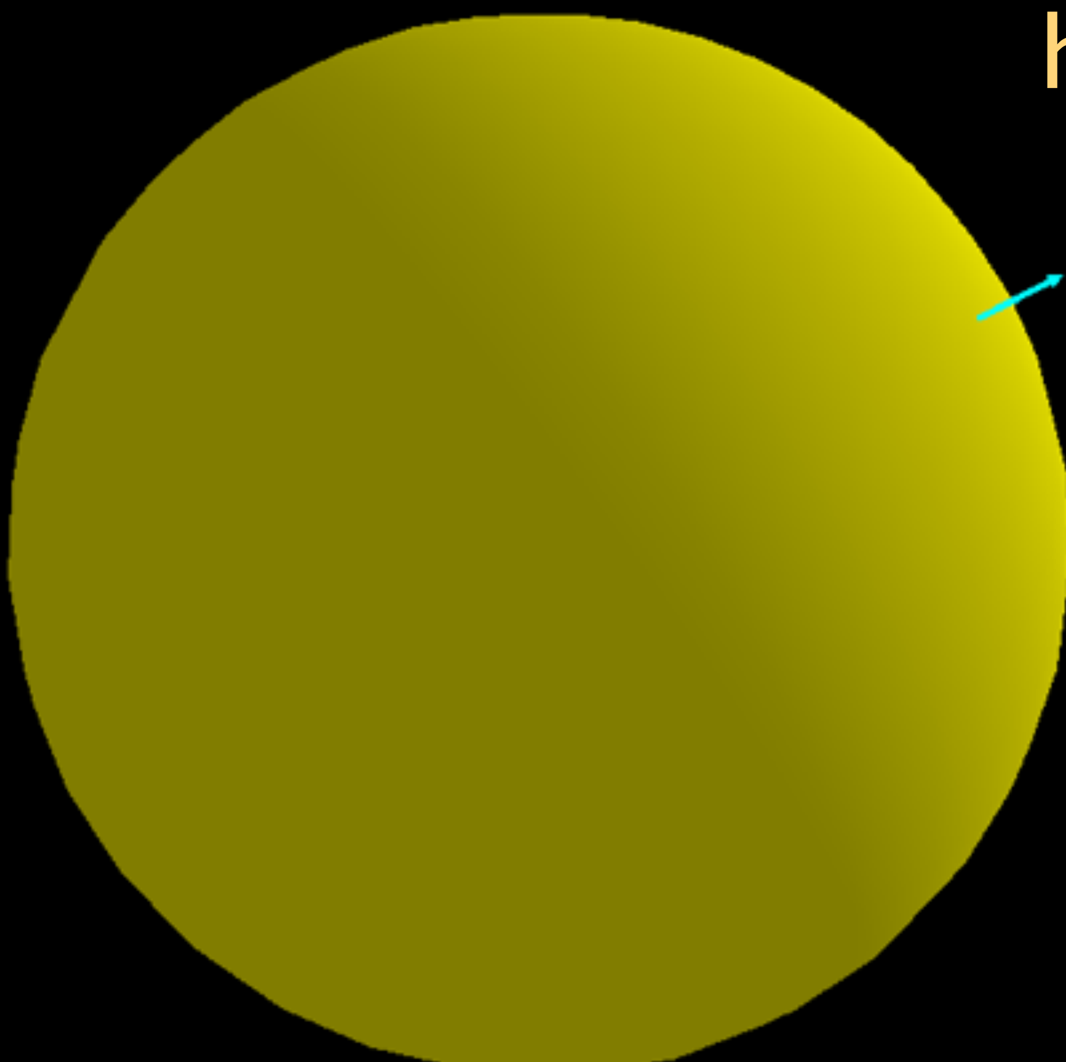
$$\psi(r, E, \vartheta)$$

energy

previous assumptions +
Azimuthal symmetry around any radial direction

(1+1)D
Single-Angle

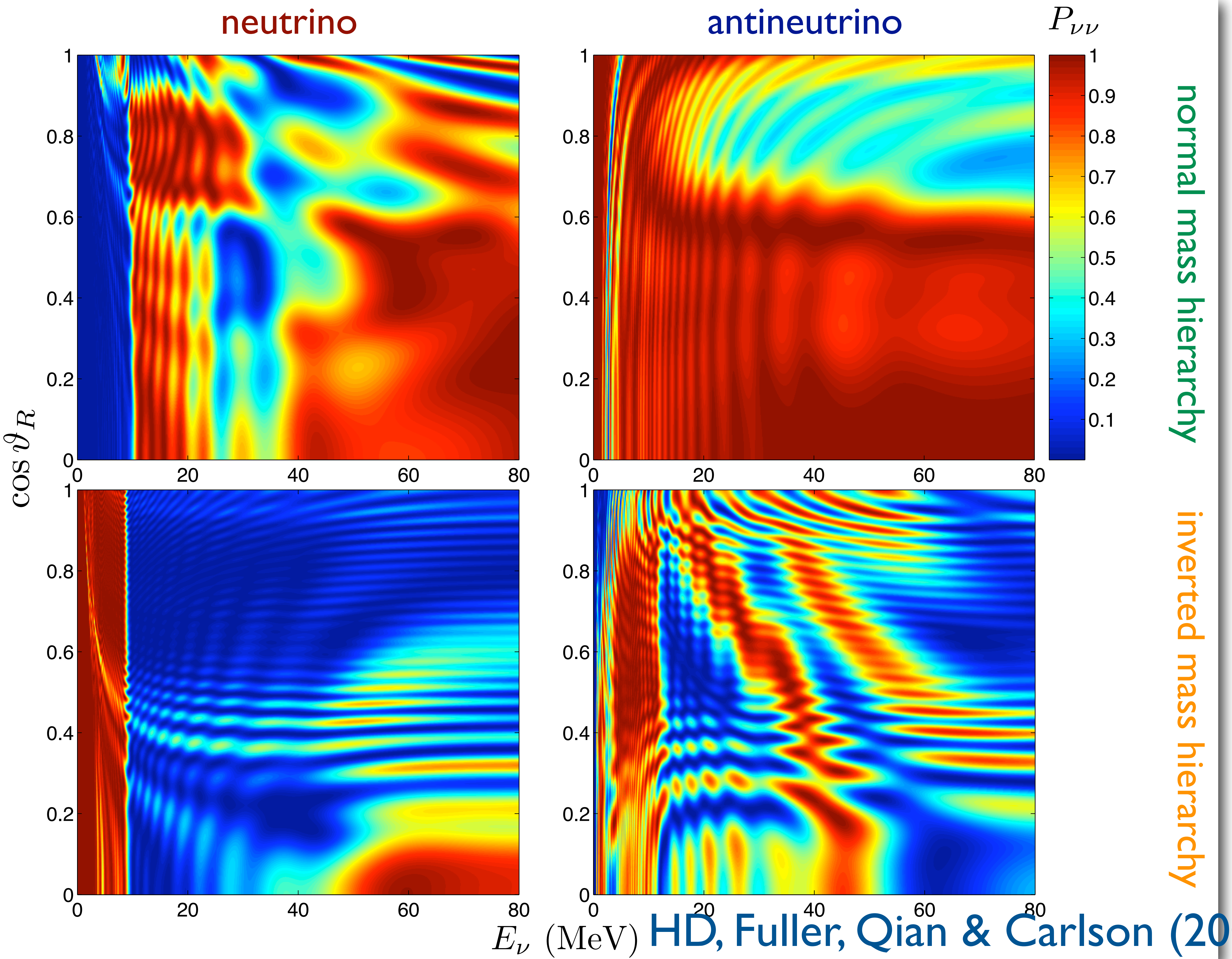
Equivalent to an expanding homogeneous neutrino gas



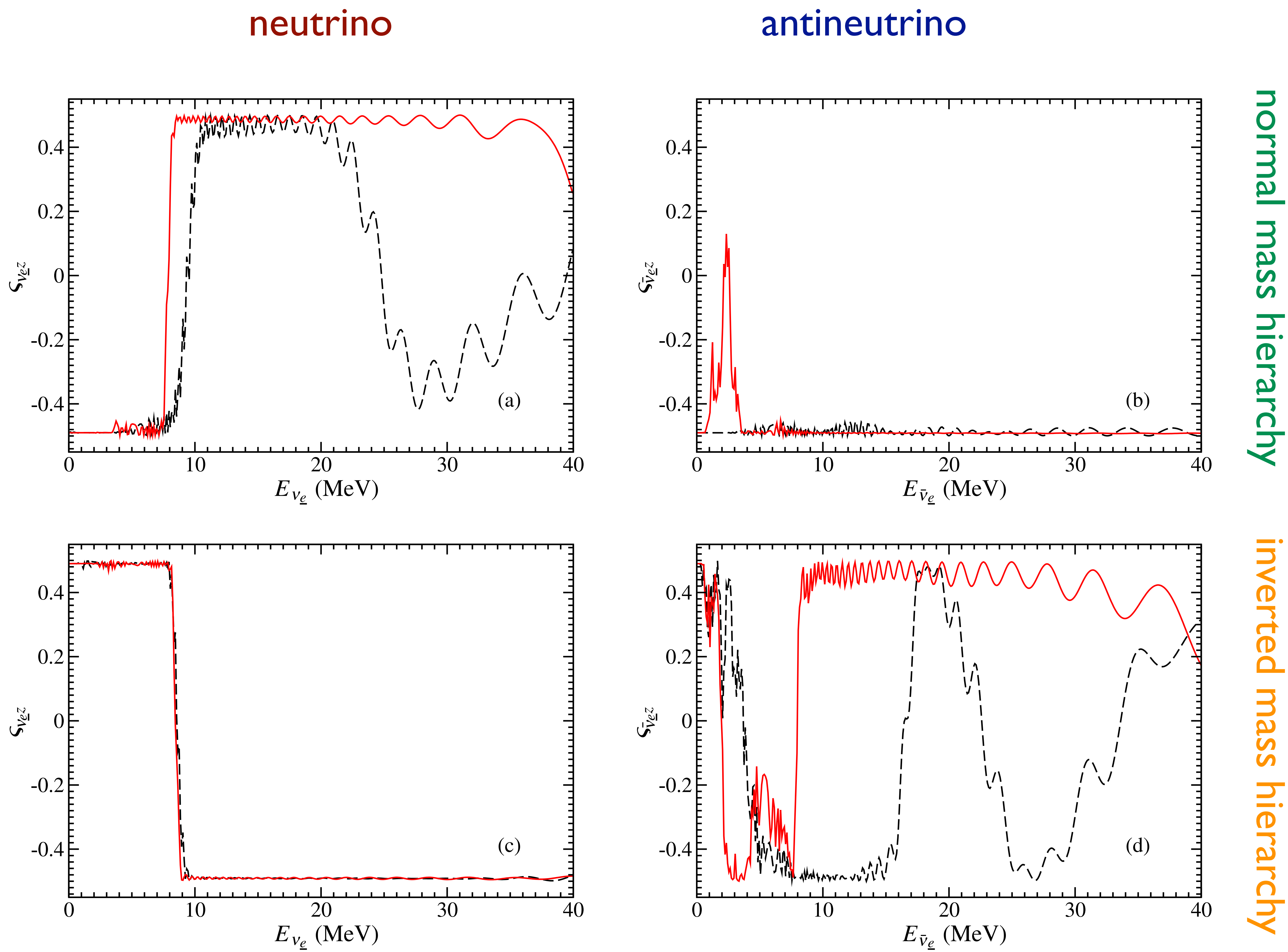
$\psi(r, E)$

↑ energy

previous assumptions +
Trajectory independent neutrino flavor evolution



HD, Fuller, Qian & Carlson (2006)



HD, Fuller, Qian & Carlson (2006)

Neutrino Self-Coupling

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}|\psi_{\nu,\mathbf{p}}\rangle = \hat{H}|\psi_{\nu,\mathbf{p}}\rangle$$

mass squared matrix → M^2

electron density → n_e

$$\mathrm{H} = \frac{\mathrm{M}^2}{2E} + \sqrt{2}G_\mathrm{F}\,\mathrm{diag}[n_e, 0, 0] + \mathrm{H}_{\nu\nu}$$

neutrino energy → $2E$

ν–ν forward scattering (self-coupling) → $\mathrm{H}_{\nu\nu}$

$$\mathrm{H}_{\nu\nu} = \sqrt{2}G_\mathrm{F}\int \mathrm{d}\mathbf{p}'(1-\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')(\rho_{\mathbf{p}'} - \bar{\rho}^*_{\mathbf{p}'})$$

Tools & Toy Models

Vacuum Oscillations

neutrinos are generated/detected in flavor states

neutrino mass eigenstates ≠ neutrino flavor states

$$|\nu_1\rangle = \cos\theta_{\rm v}|\nu_e\rangle + \sin\theta_{\rm v}|\nu_\mu\rangle \quad \text{with mass } m_1$$

$$|\nu_2\rangle = -\sin\theta_{\rm v}|\nu_e\rangle + \cos\theta_{\rm v}|\nu_\mu\rangle \quad \text{with mass } m_2$$

vacuum mixing angle

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-\omega\cos 2\theta_{\rm v} & \omega\sin 2\theta_{\rm v}\\ \omega\sin 2\theta_{\rm v} & \omega\cos 2\theta_{\rm v}\end{bmatrix}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix}$$

vac. osc. freq. $\omega = \dfrac{\delta m^2}{2E_\nu}$

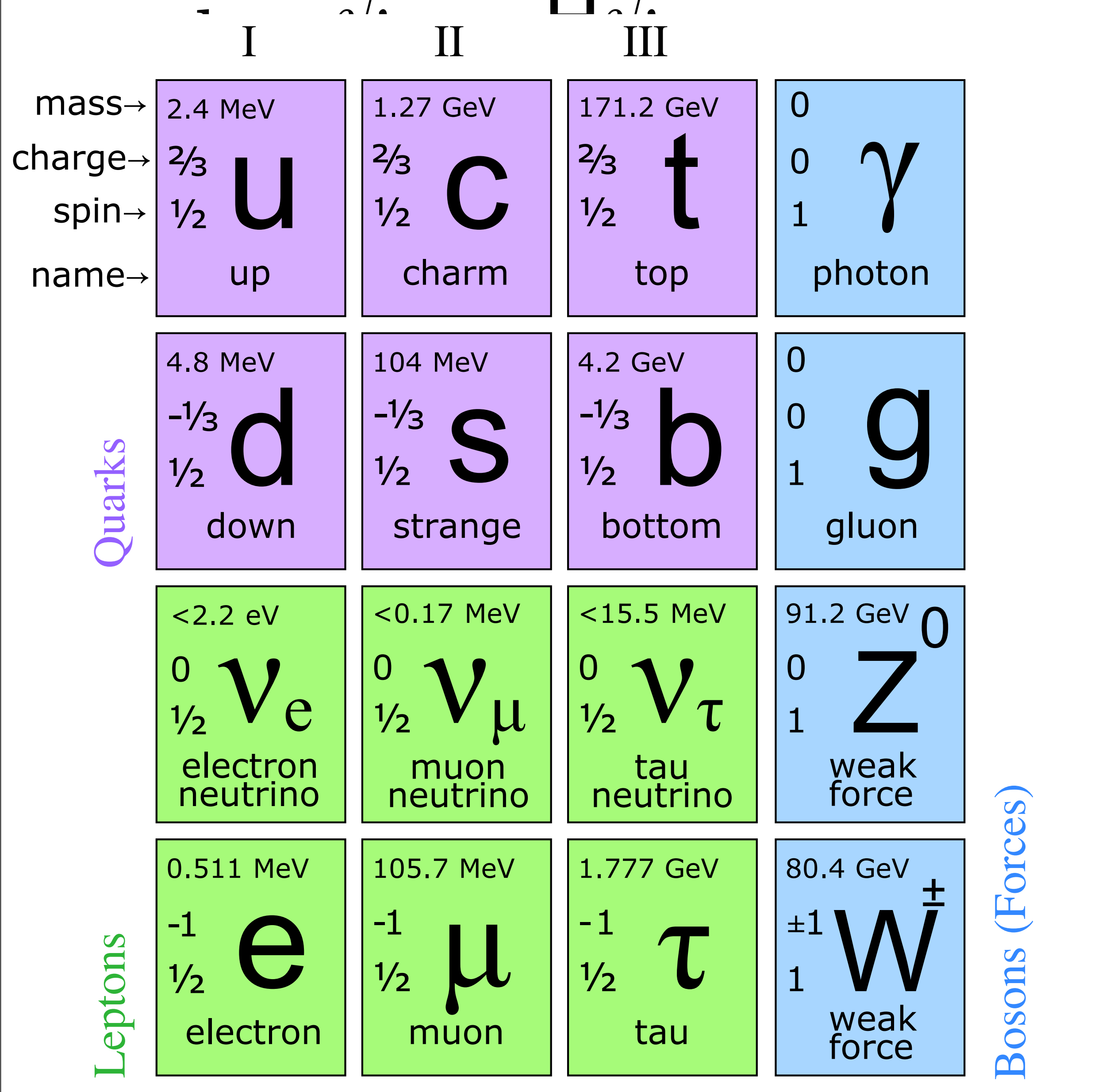
$$\delta m^2 = m_2^2 - m_1^2$$

Neutrino Flavor Isospin

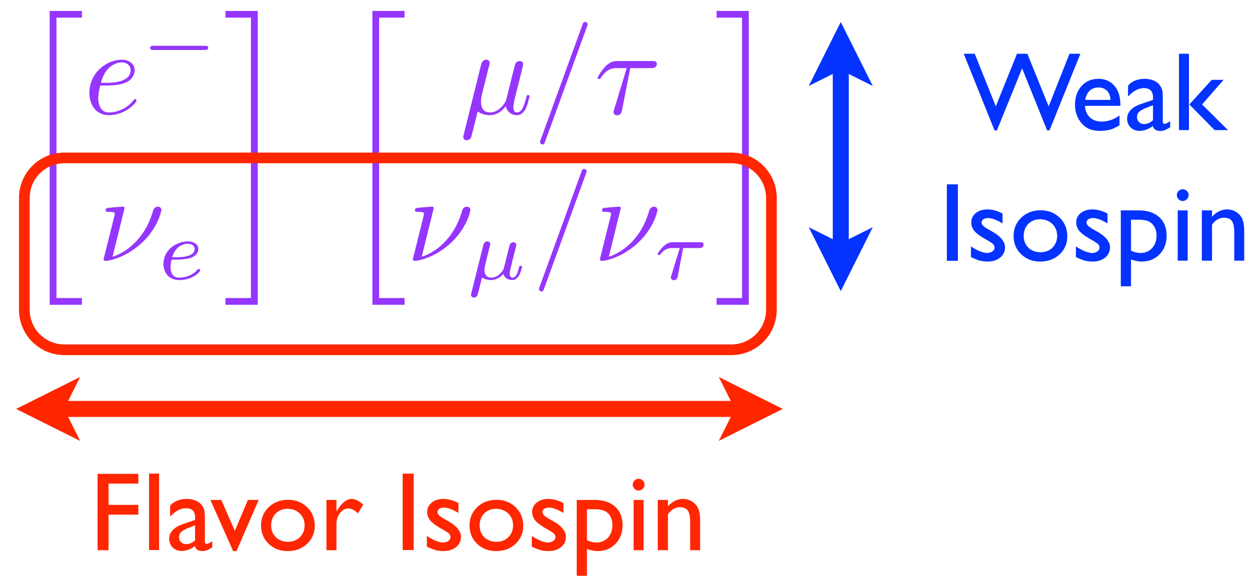
Two-component system ⟷ spin-1/2

2×2 Hermitian matrix $\mathsf{H} = H_0 \mathbb{1} + \mathbf{H} \cdot \boldsymbol{\sigma}$

Neutrino Flavor Isospin



Wikimedia: Standard Model of Elementary Particles



...avor τ'-flavor maximally mixed

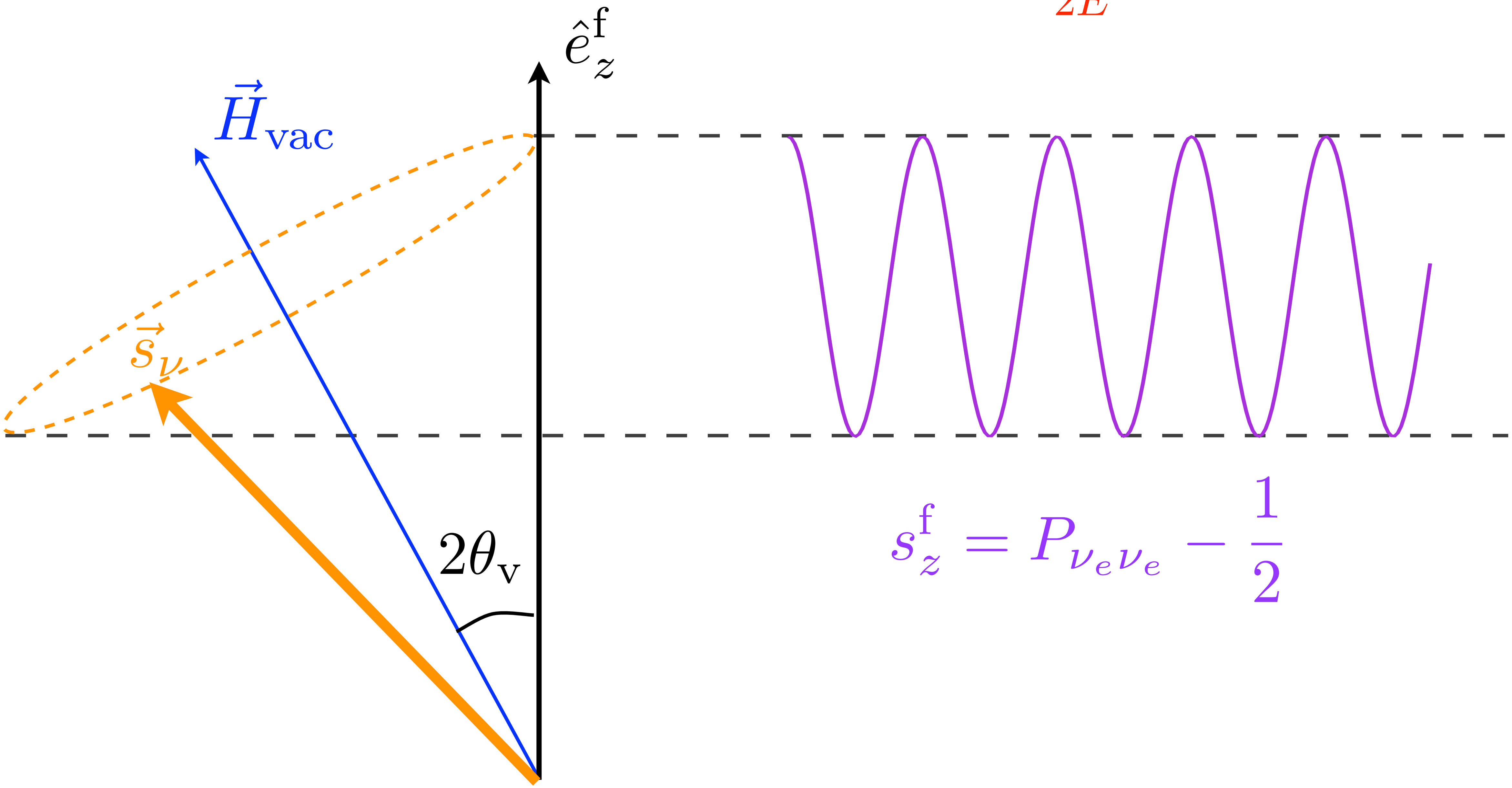
$\downarrow$ $\rightarrow$

$\uparrow$ $\rightarrow$

Vacuum Oscillations Again

$$\vec{H} = \omega \vec{H}_{\rm vac}$$

$$\vec{H}_{\rm vac} \equiv -\hat{e}_x^{\rm f} \sin 2\theta_{\rm v} + \hat{e}_z^{\rm f} \cos 2\theta_{\rm v} \qquad \omega \equiv \pm \frac{\delta m^2}{2E}$$



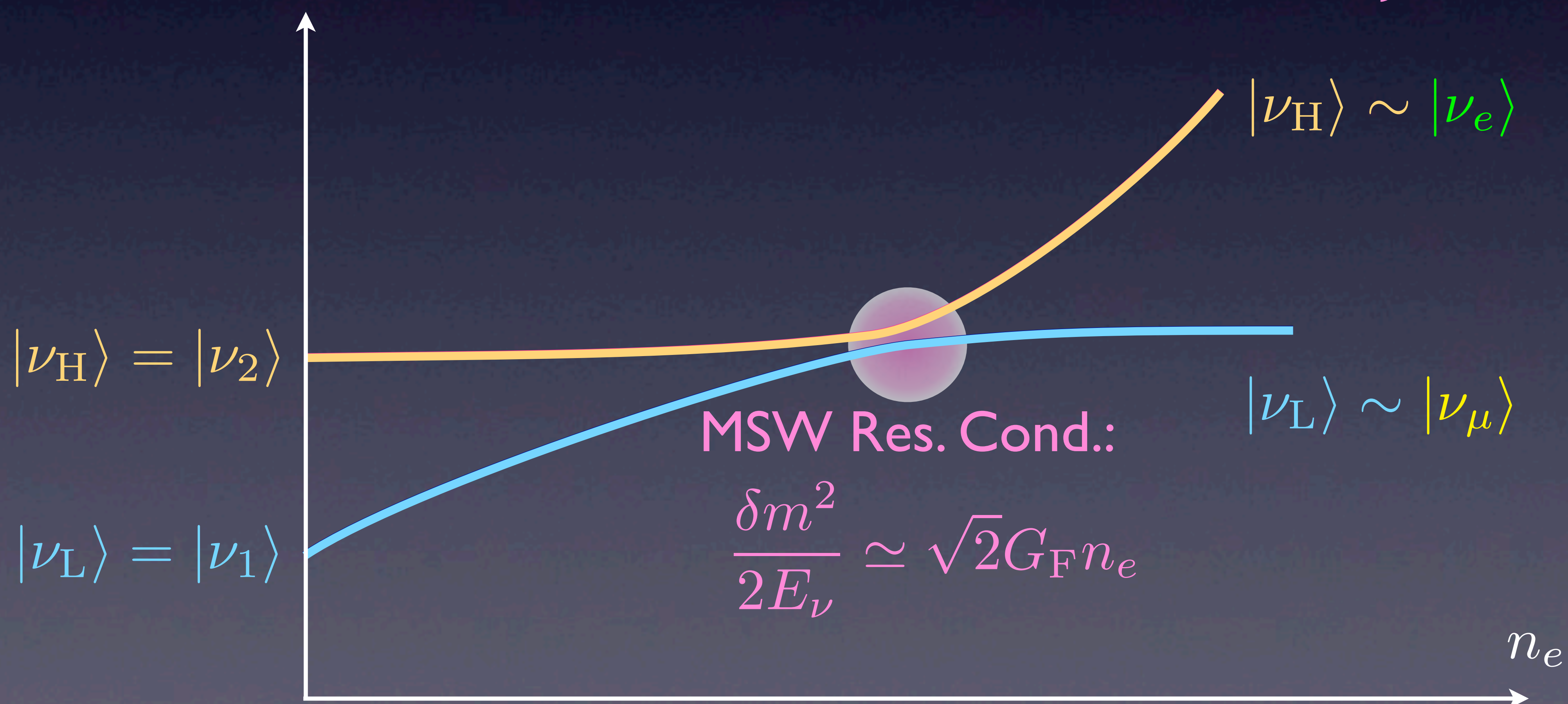
$$s_z^{\rm f} = P_{\nu_e \nu_e} - \frac{1}{2}$$

MSW Effect

electron number density

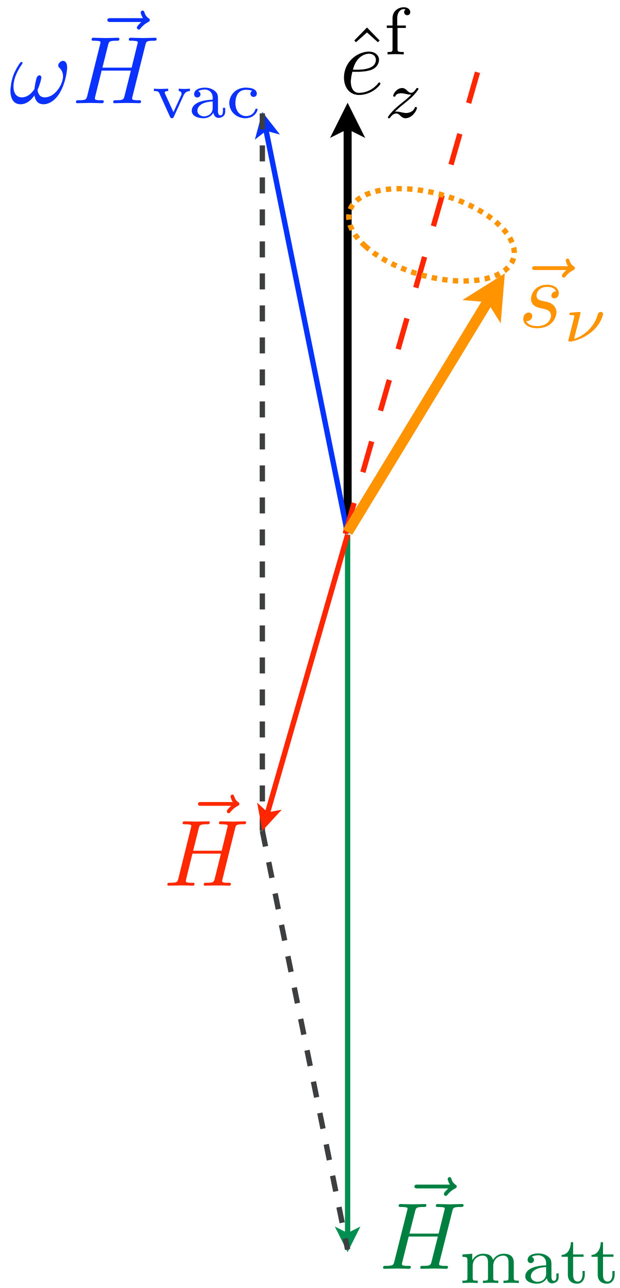
$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2\sqrt{2}G_{\mathrm{F}}n_e - \omega\cos 2\theta_{\mathrm{v}} & \omega\sin 2\theta_{\mathrm{v}}\\ \omega\sin 2\theta_{\mathrm{v}} & \omega\cos 2\theta_{\mathrm{v}}\end{bmatrix}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix}$$

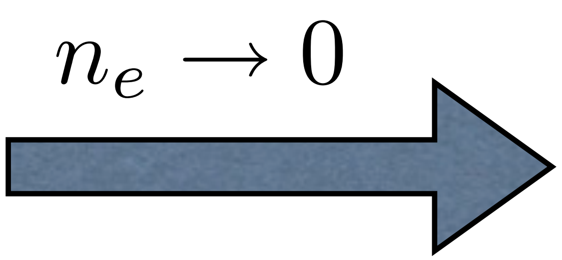
vac. osc. freq. $\omega = \frac{\delta m^2}{2E_\nu}$

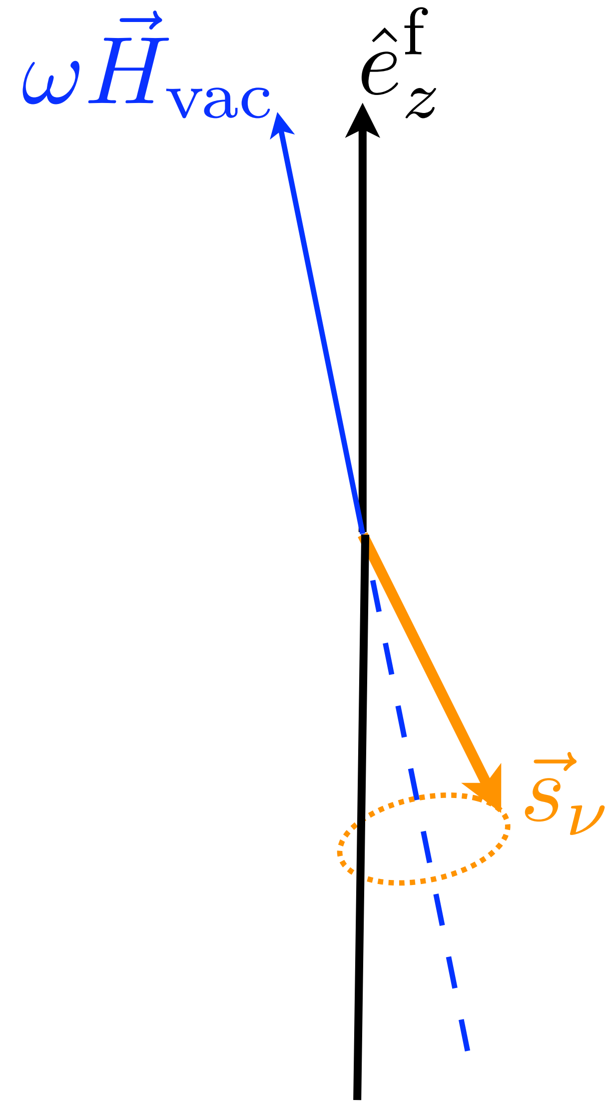


MSW Again

$$\vec{H} = \omega \vec{H}_{\rm vac} + \vec{H}_{\rm matt} \qquad \vec{H}_{\rm matt} \equiv -\hat{e}_z^{\rm f} \sqrt{2} G_{\rm F} n_e$$







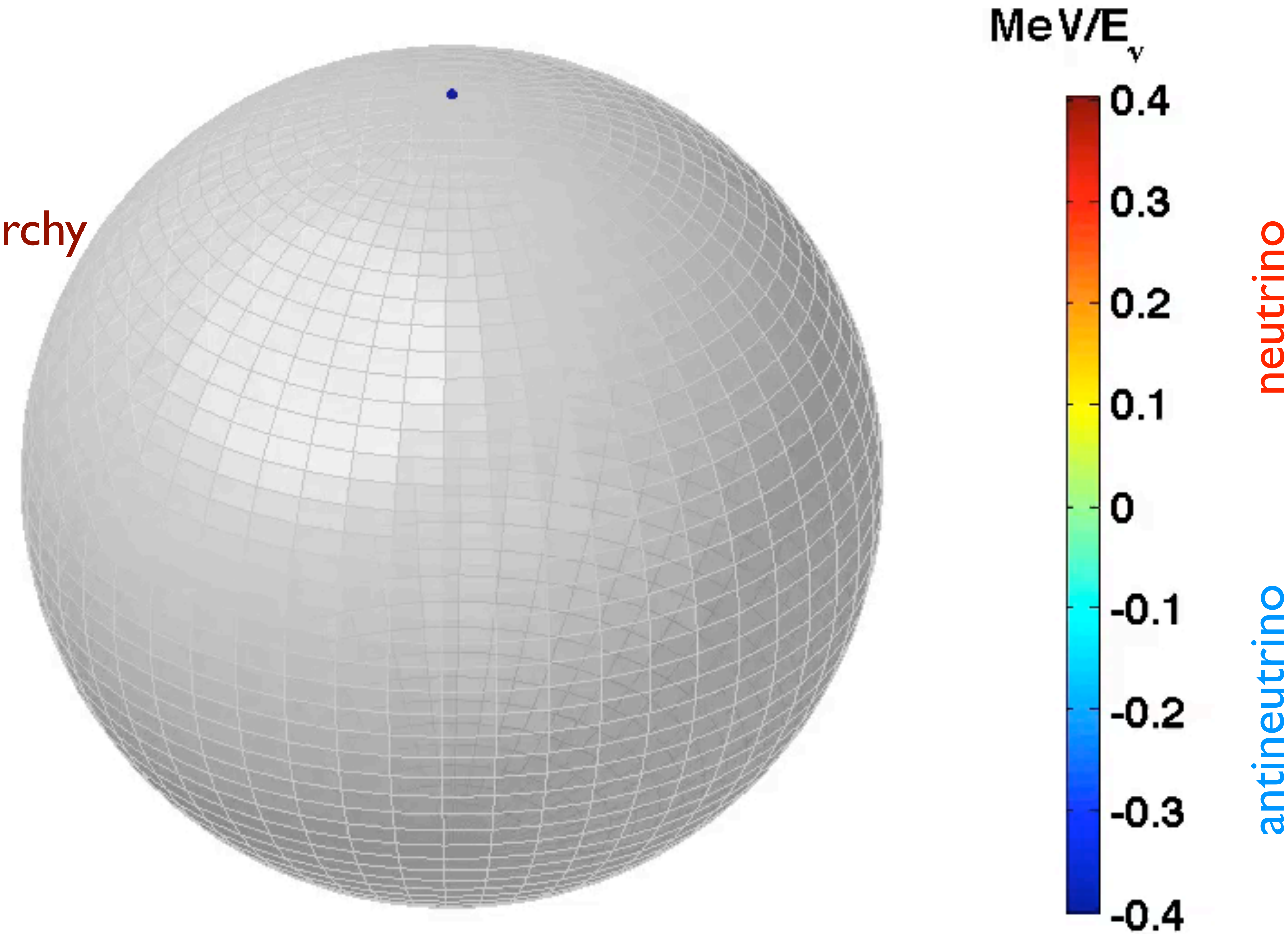
$$\omega = \frac{\delta m^2}{2E}, \; \delta m^2 > 0$$

neutrino, normal hierarchy

MSW Mechanism

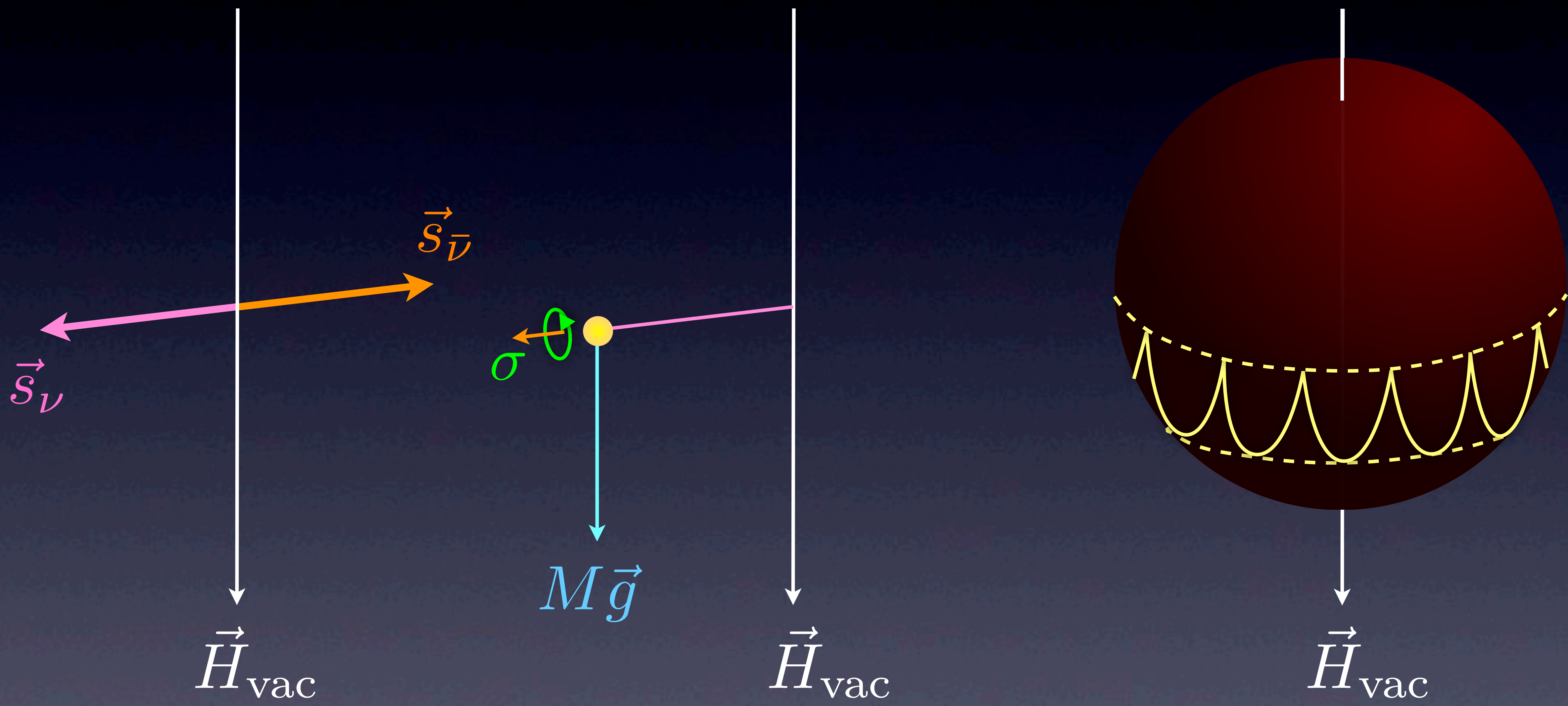
$\delta m^2 < 0$

inverted hierarchy



Bipolar System

Mono-energetic ν-$\bar{\nu}$ gas

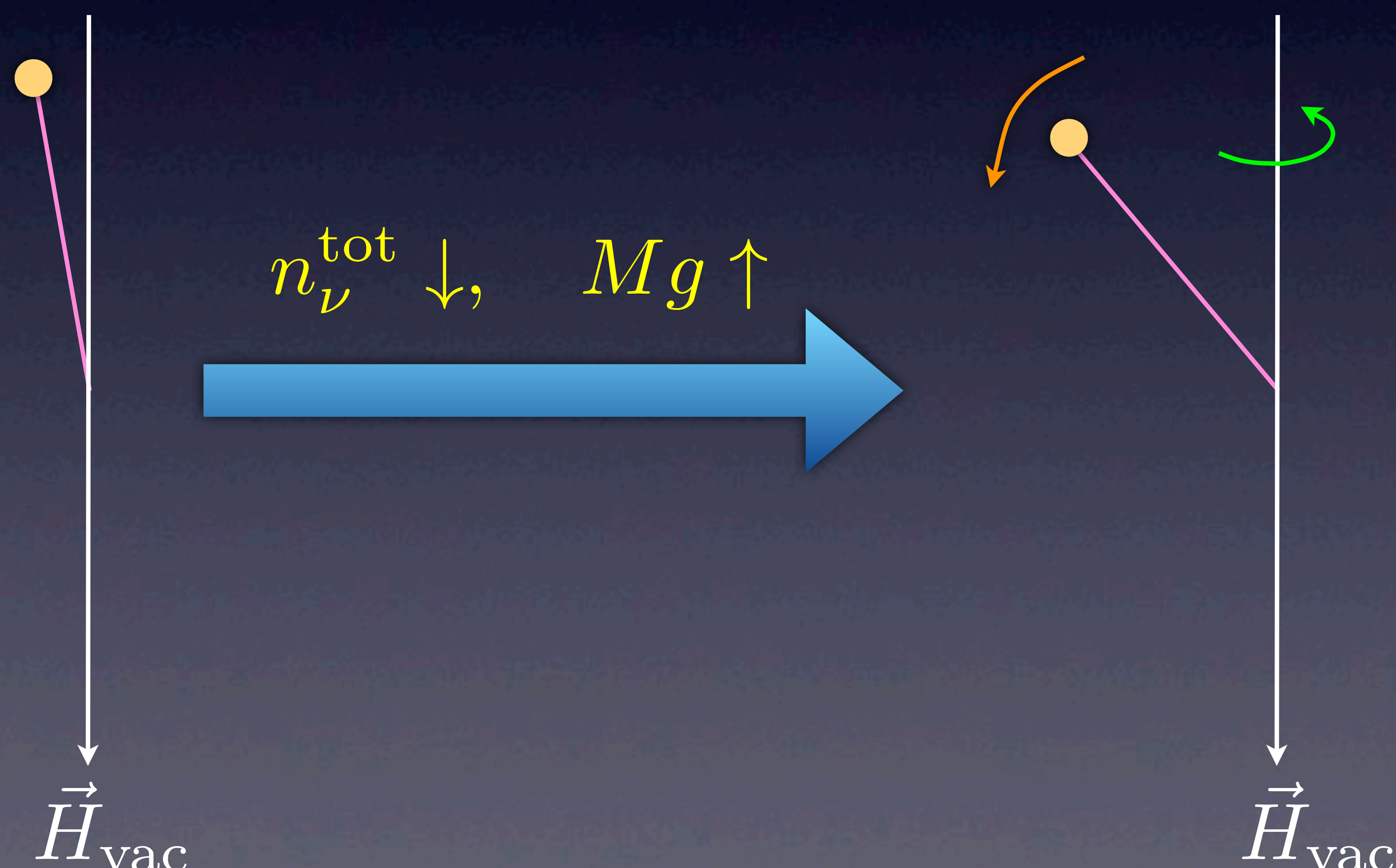


$$\sigma \sim \frac{n_\nu - n_{\bar{\nu}}}{n_\nu + n_{\bar{\nu}}} \qquad M\vec{g} \sim \frac{\vec{H}_{\rm vac}}{n_\nu + n_{\bar{\nu}}}$$

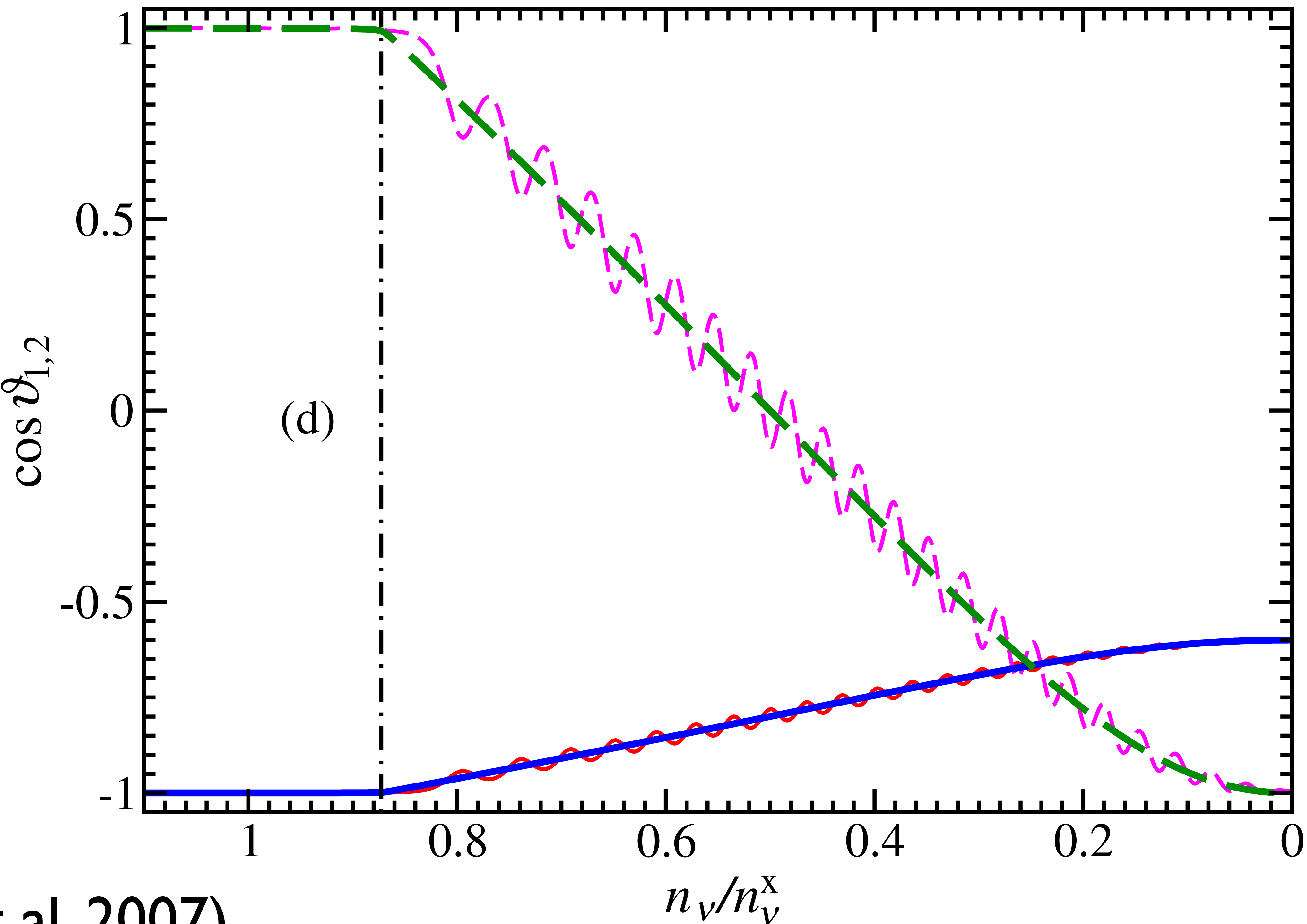
(Hannestad et al, 2006; HD et al, 2007)

Bipolar System

Inverted Mass Hierarchy



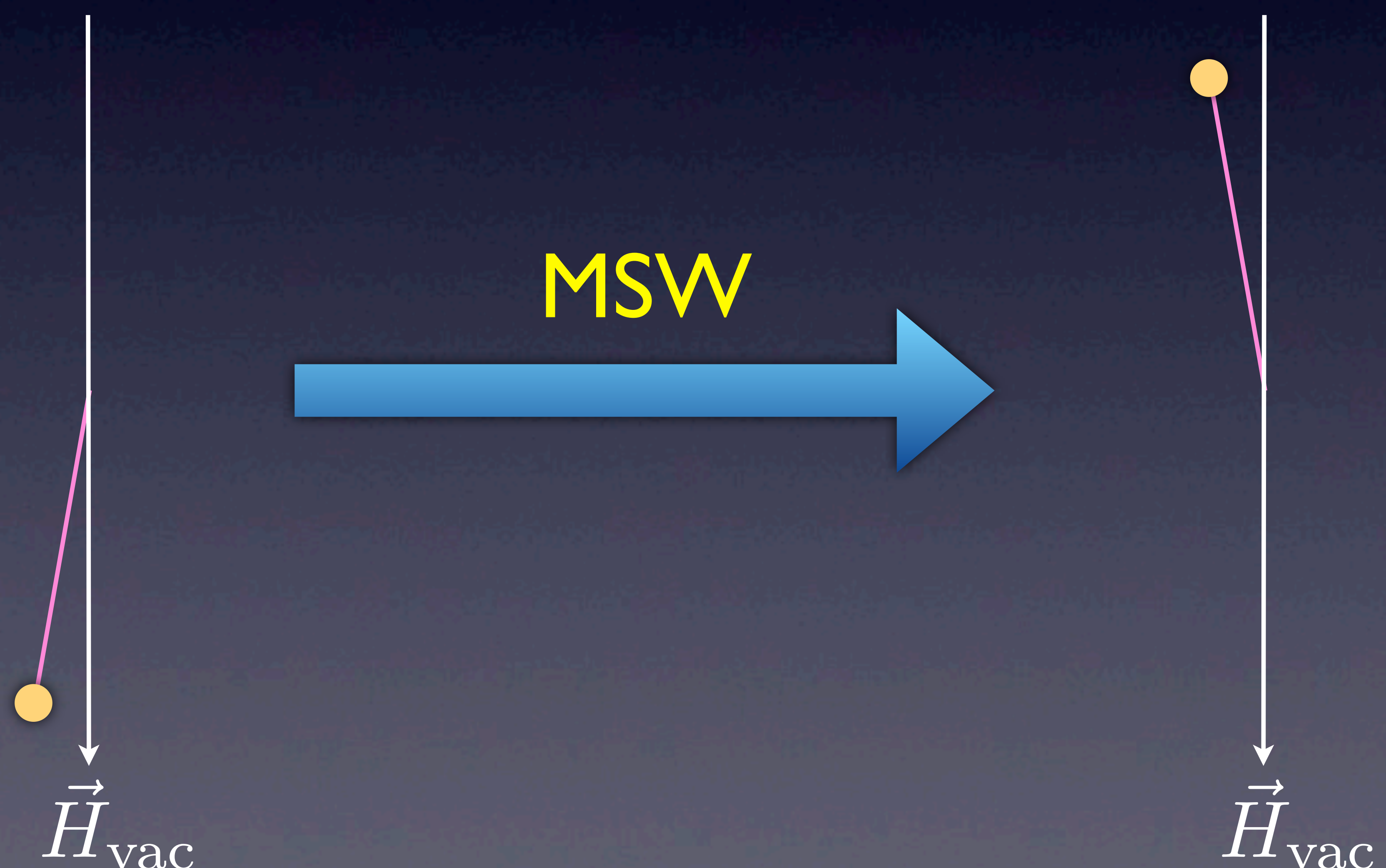
Bipolar System



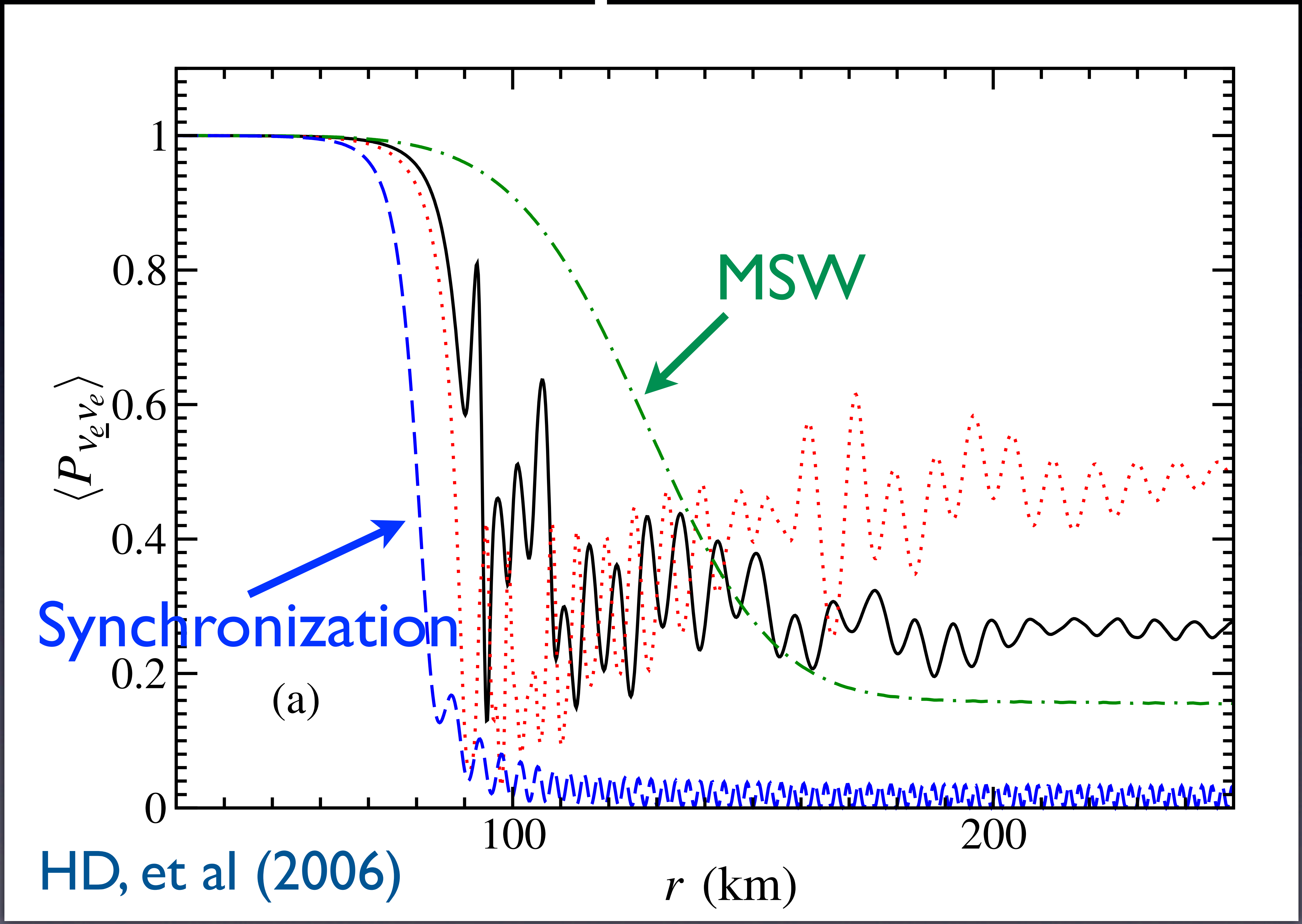
(HD et al, 2007)

Bipolar System

Normal Mass Hierarchy

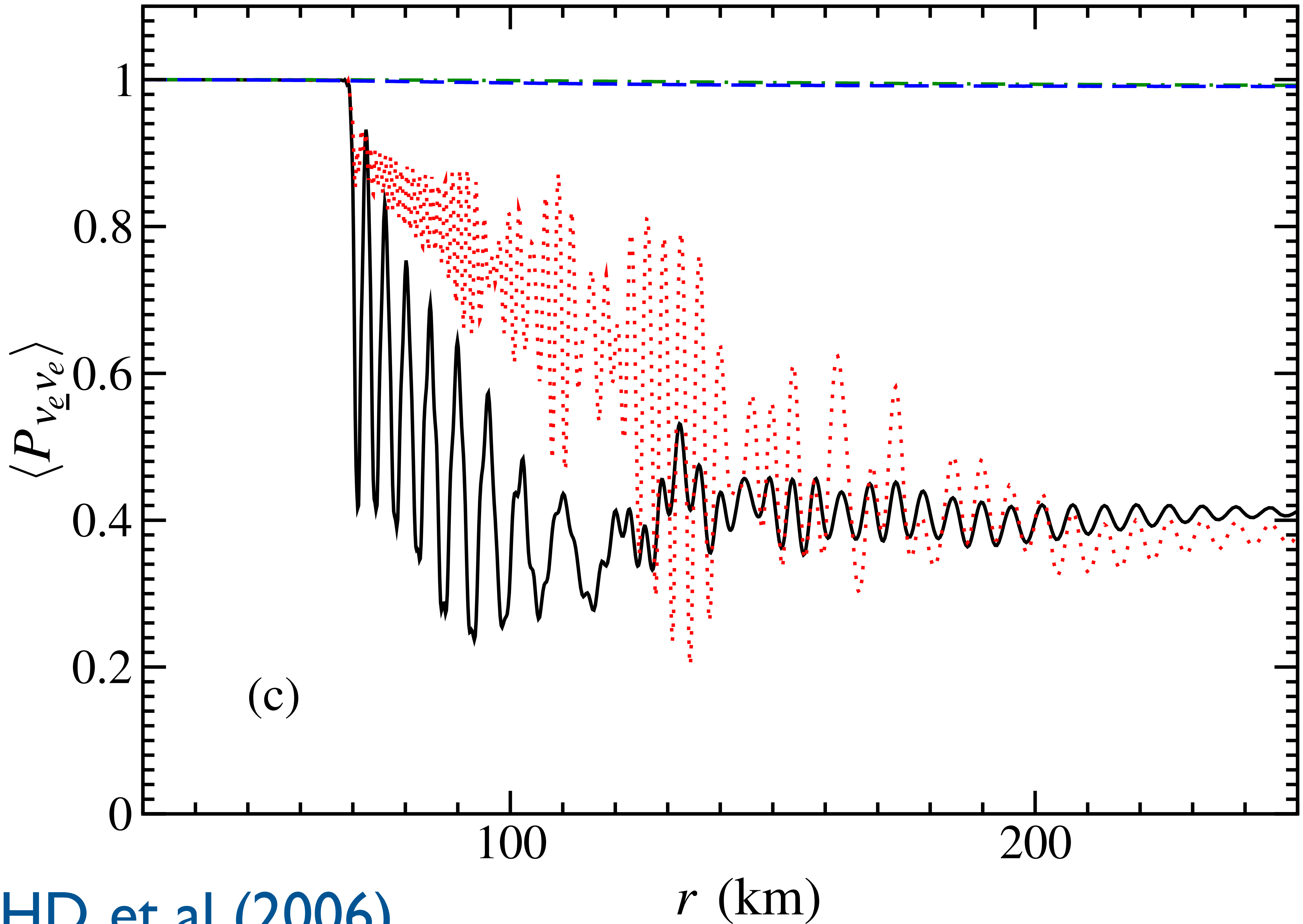


Comparison



MSW
Synchronization
HD, et al (2006)
(a)
$\langle P_{\nu_e \nu_e} \rangle$
r (km)
0
0.2
0.4
0.6
0.8
1
100
200

Comparison



HD, et al (2006)

Homogeneous Gas

$$\frac{\mathrm{d}}{\mathrm{d}r}\vec{s}_\omega = \vec{s}_\omega \times \vec{H}_\omega$$

$$\vec{H}_\omega = \vec{H}_{\mathrm{vac}} + \vec{H}_{\mathrm{matt}} + \vec{H}_{\nu\nu}$$

$$\vec{H}_{\mathrm{vac}} = \omega \hat{e}_z^{\mathrm{v}}$$

Depend on neutrino energy;
disrupt collective oscillations

$$\vec{H}_{\mathrm{matt}} = -\sqrt{2} G_{\mathrm{F}} n_e \hat{e}_z^{\mathrm{f}}$$

Independent of neutrino energy;
"Ignored" for collective oscillations

$$\vec{H}_{\nu\nu} = -2\sqrt{2} G_{\mathrm{F}} n_\nu^{\mathrm{tot}} \int_{-\infty}^{\infty} \mathrm{d}\omega' f_{\omega'} \vec{s}_{\omega'}$$

$$= -\mu \langle \vec{s} \rangle$$

Independent of neutrino enegy;
Drive collective oscillations

avg NFIS

coupling strength

anti-ferromagnetic

NFIS distribution

Collective Oscillations

rotational symmetry of EoM

↓

collective precession of flavor isospins

↓

rotating "magnetic field"

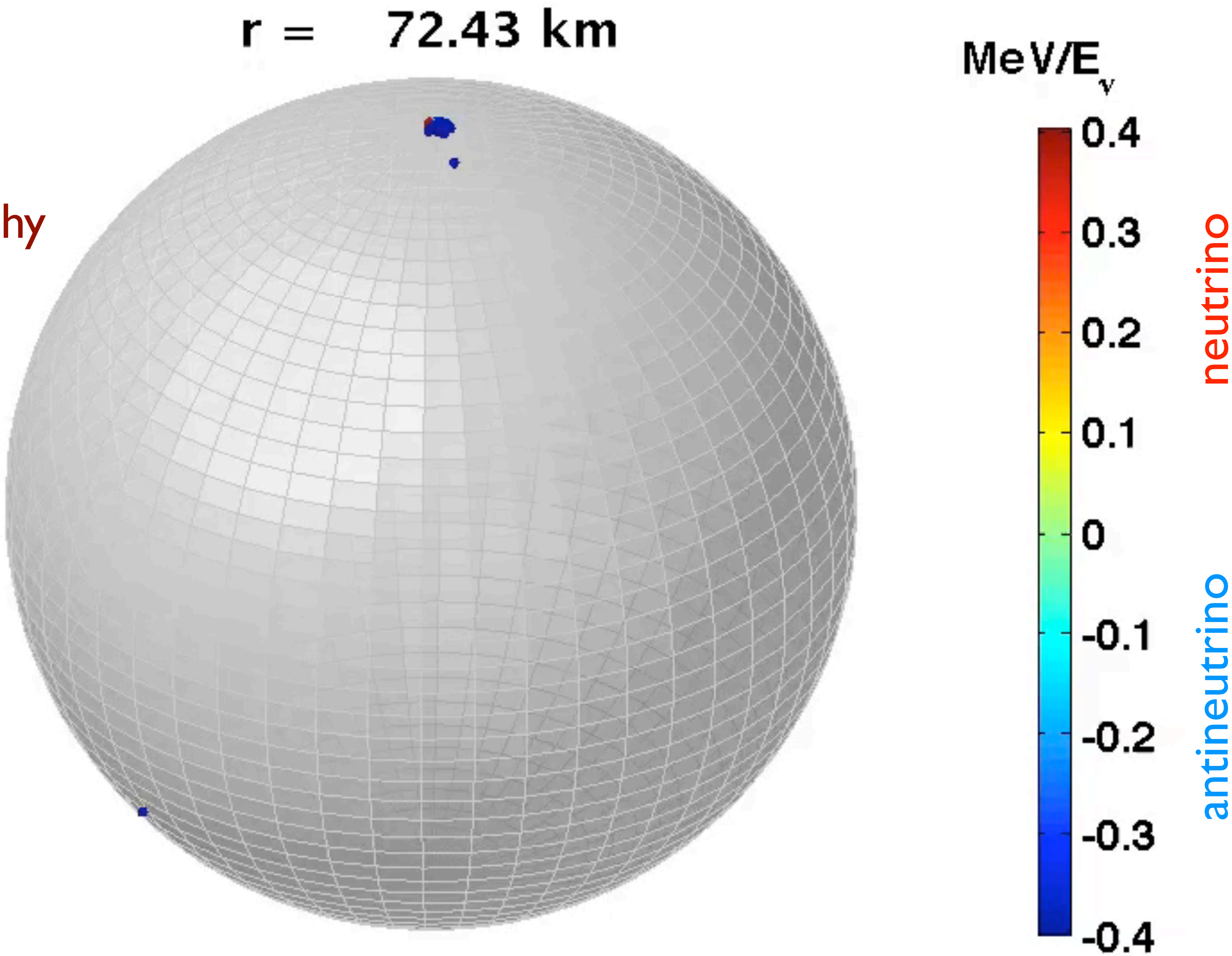
↓ magnetic spin resonance

new flavor transformation mechanism

Collective Oscillations

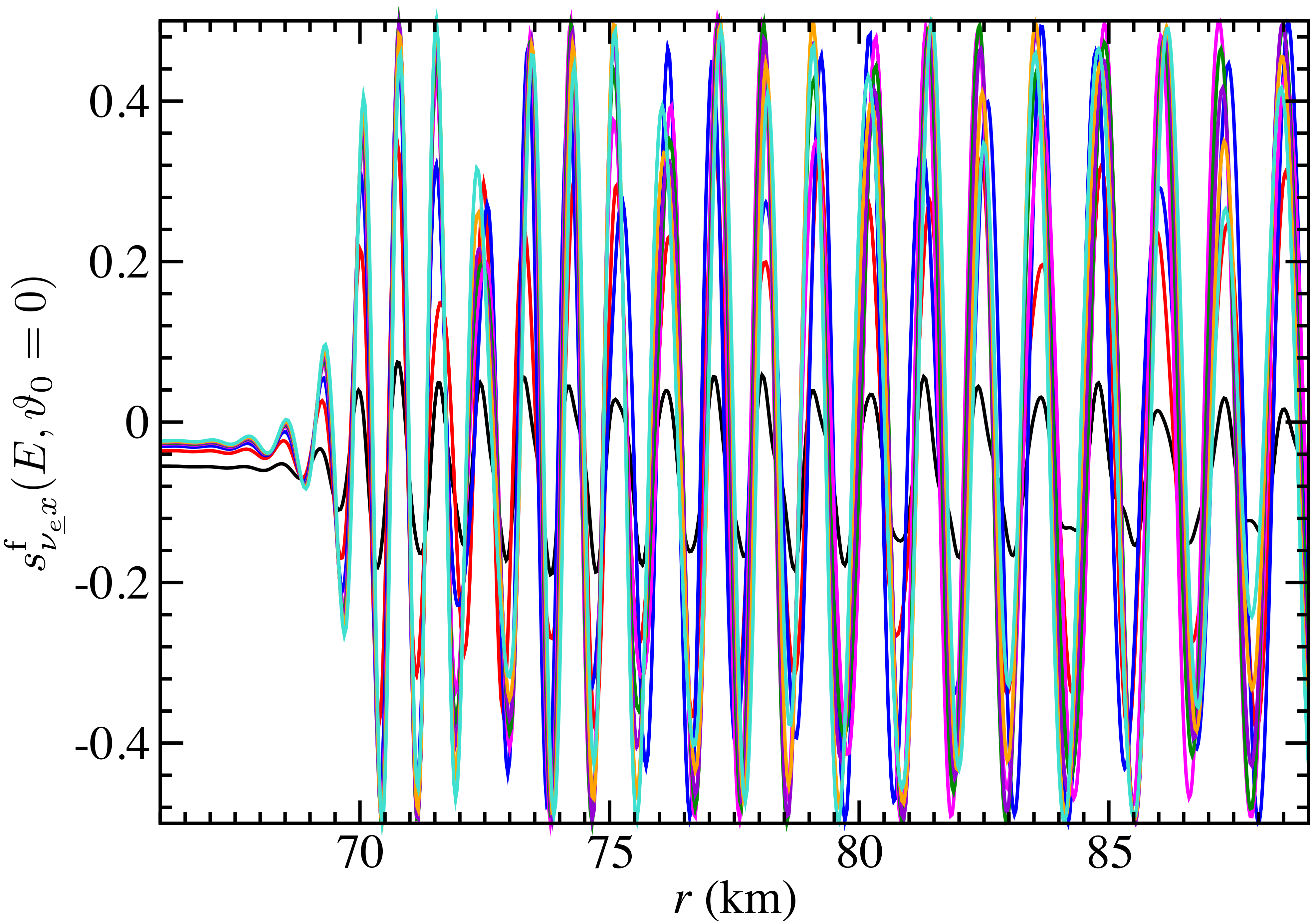
$\delta m^2 < 0$

inverted hierarchy



Multi-angle calculation

$$\delta m^2 = -3 \times 10^{-3}\,\mathrm{eV}^2 \simeq \delta m^2_{\rm atm}, \quad \theta_{\rm v} = 0.1$$



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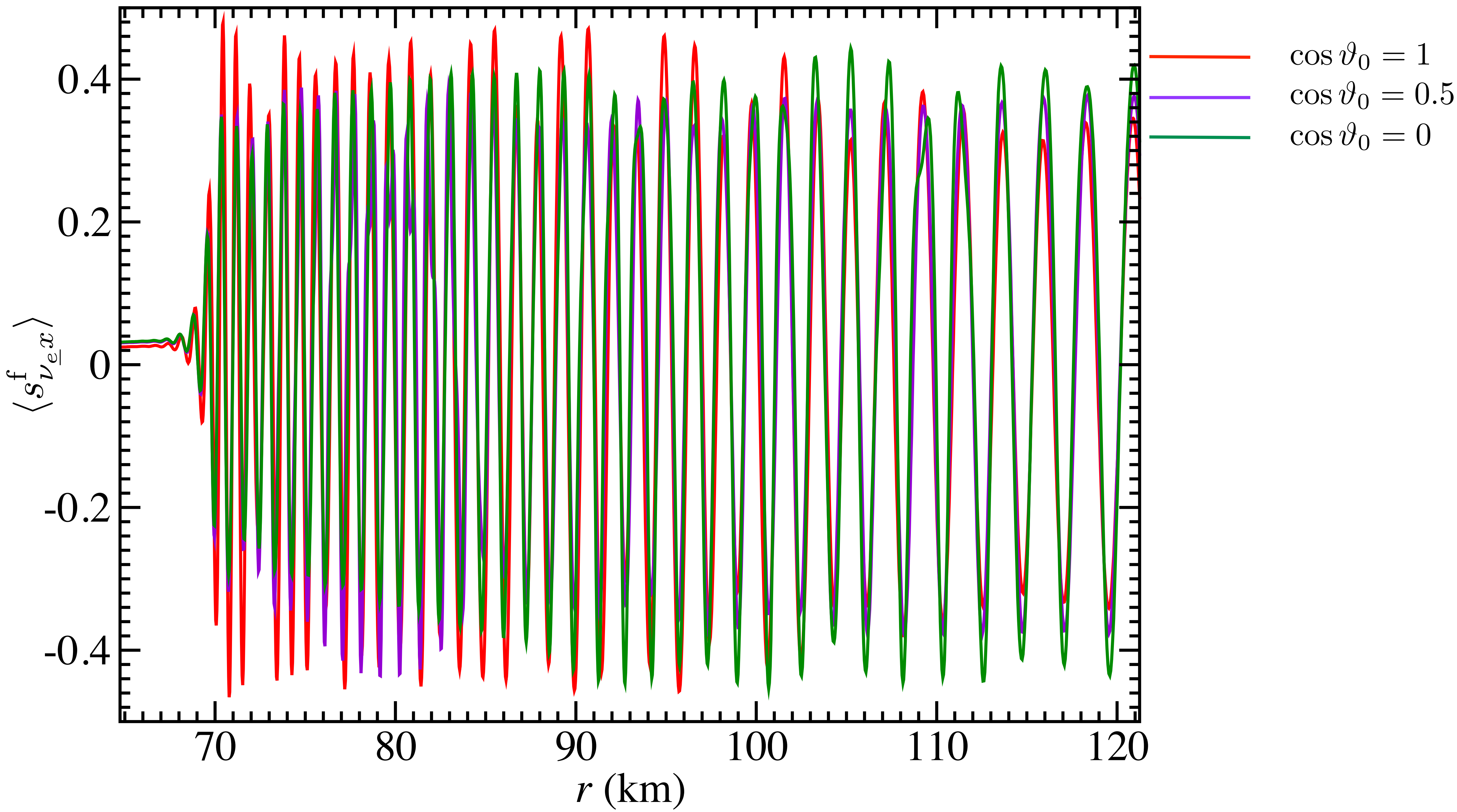
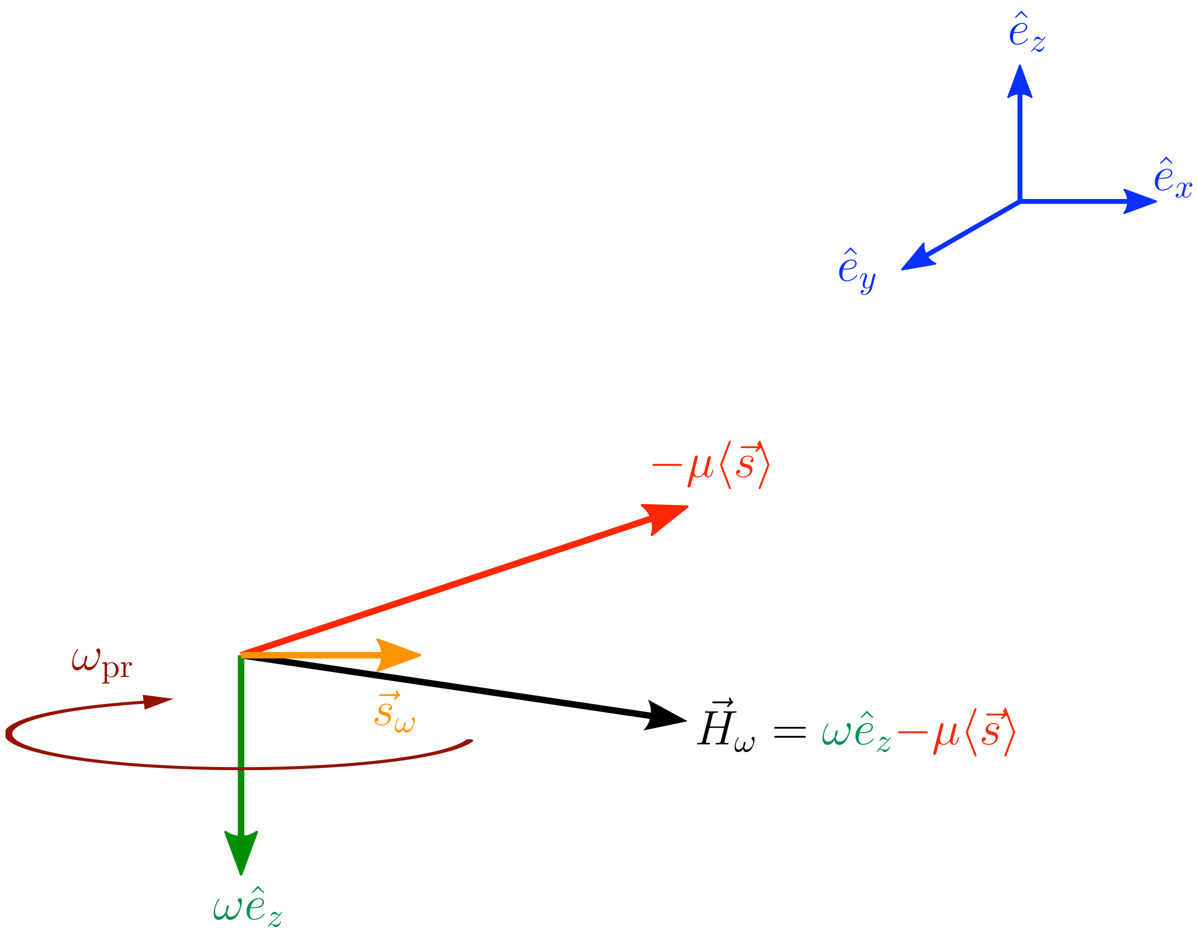


Precession Mode

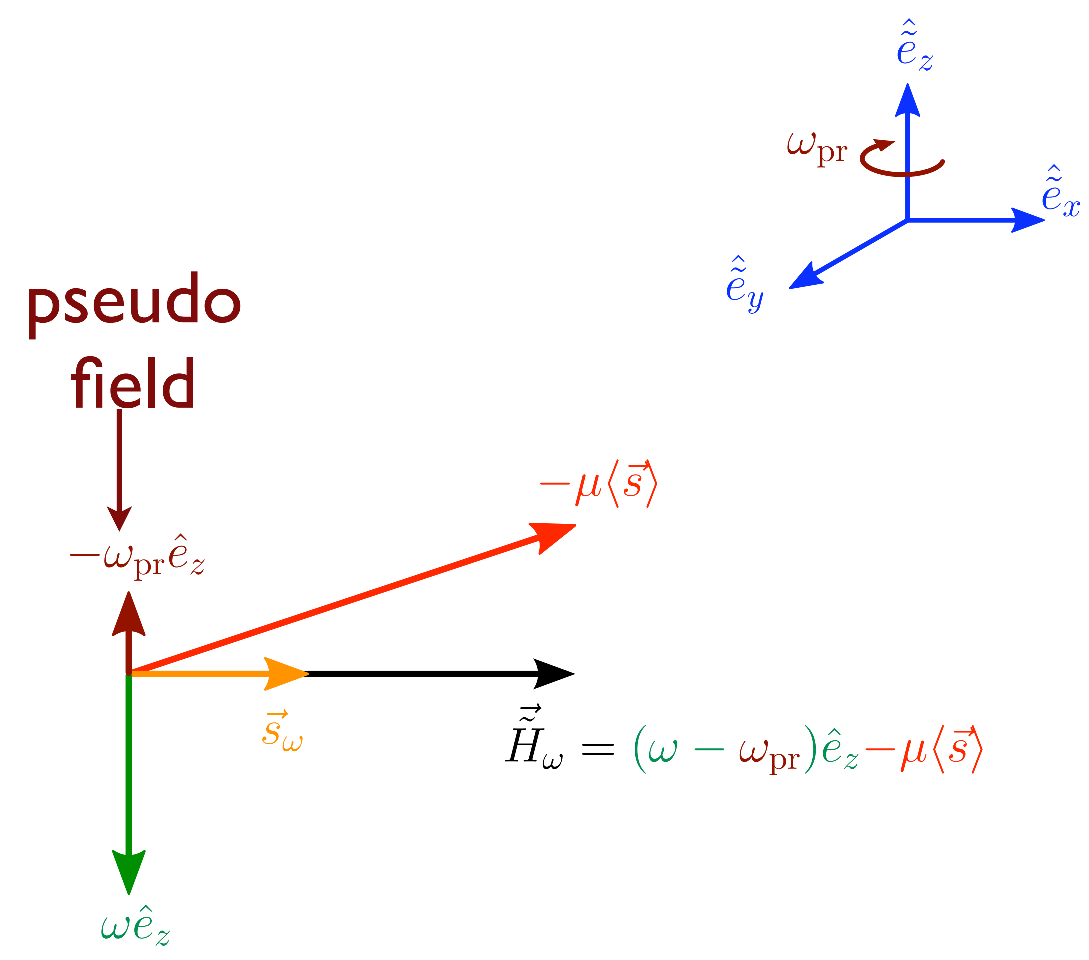
precession ansatz

all $\vec{s}_\omega$ precess about $\hat{e}_z$ with a common angular speed $\omega_{\rm pr}$

static frame



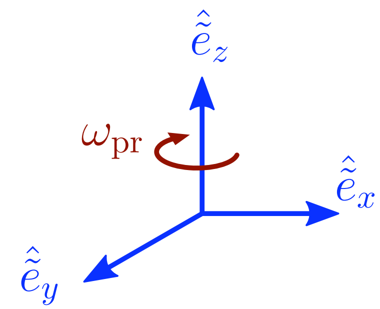
corotating frame

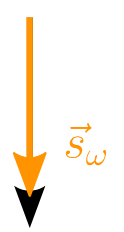


HD, Fuller, Carlson & Qian (2006)

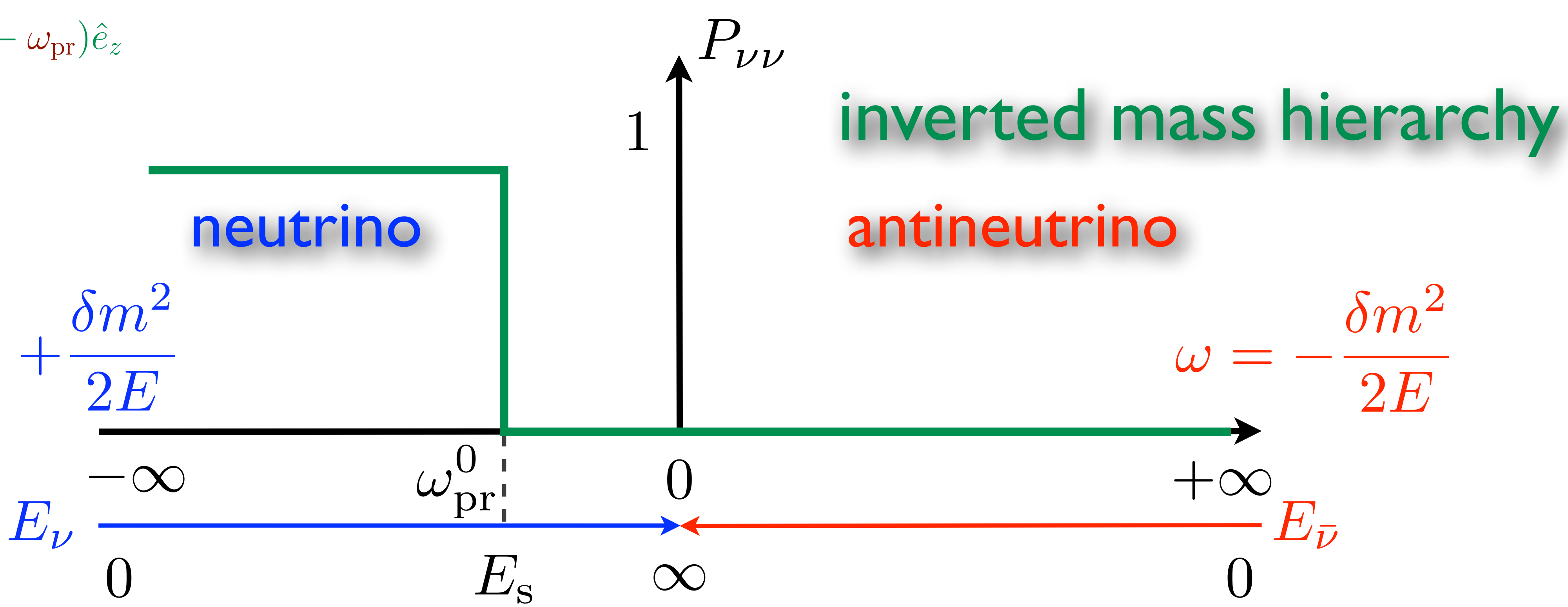
Adiabatic Process

$n_\nu^{\rm tot} \to 0$



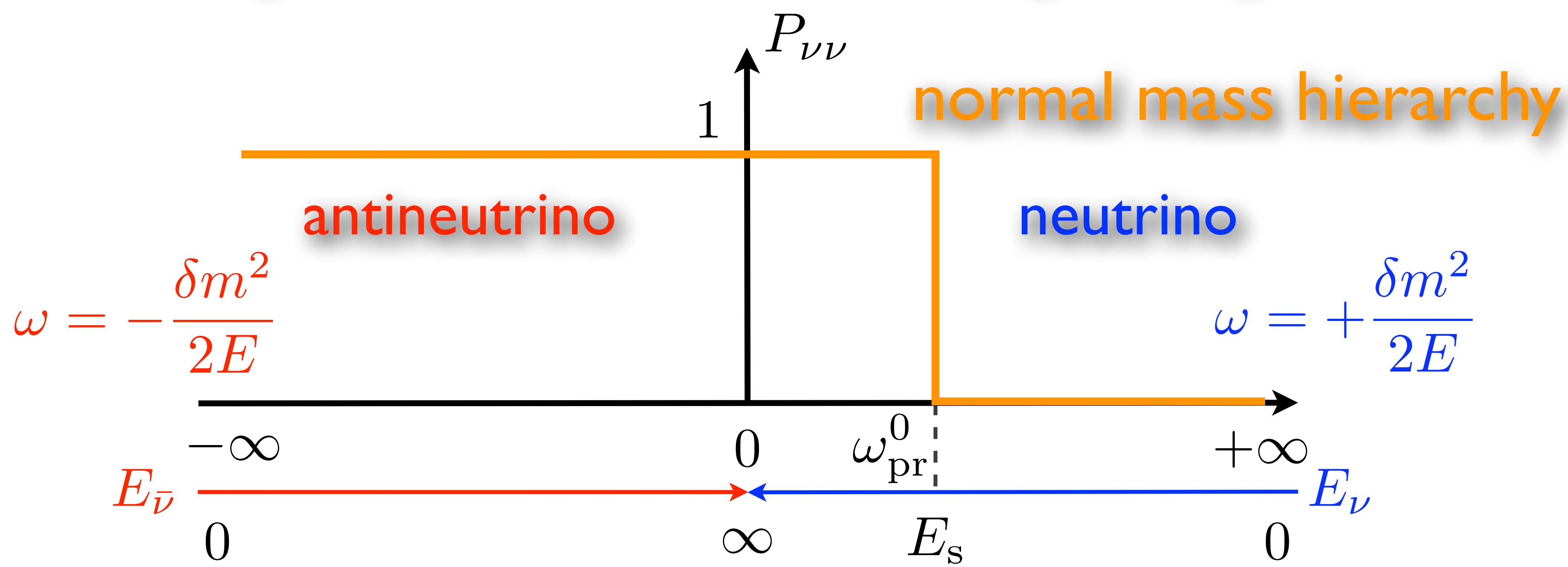


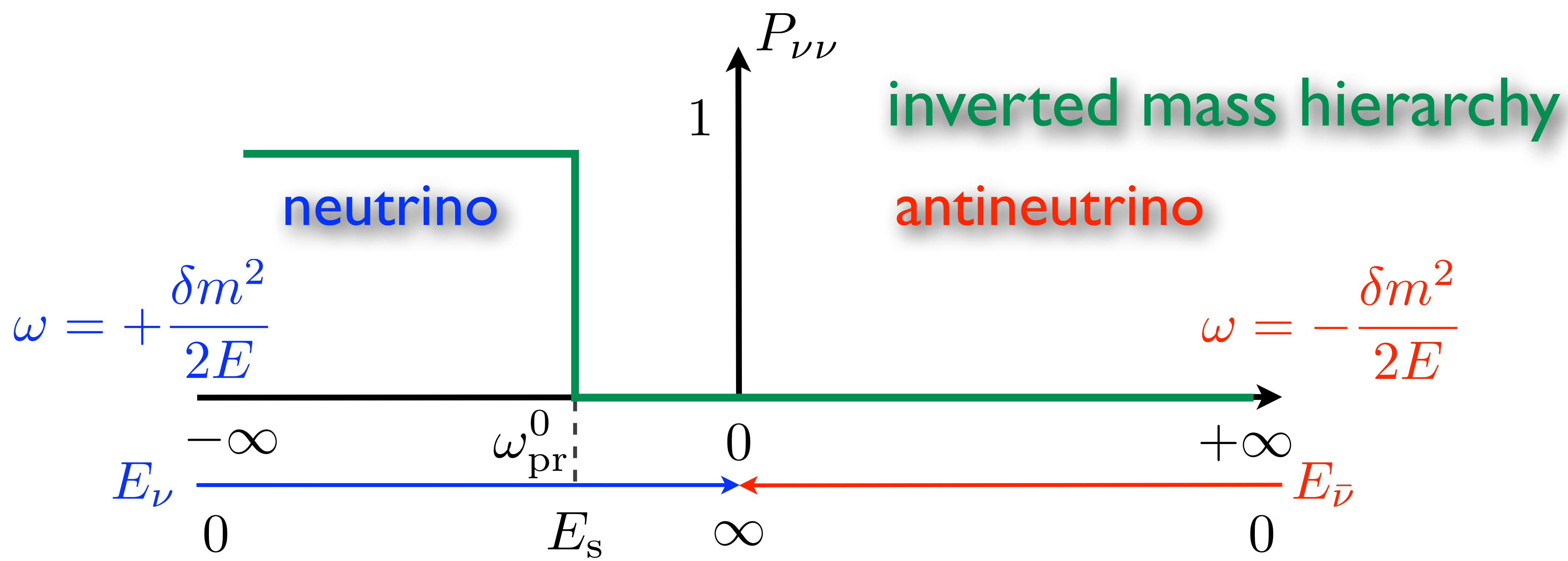
$\vec{\tilde{H}}_\omega = (\omega - \omega_{\rm pr})\hat{e}_z$

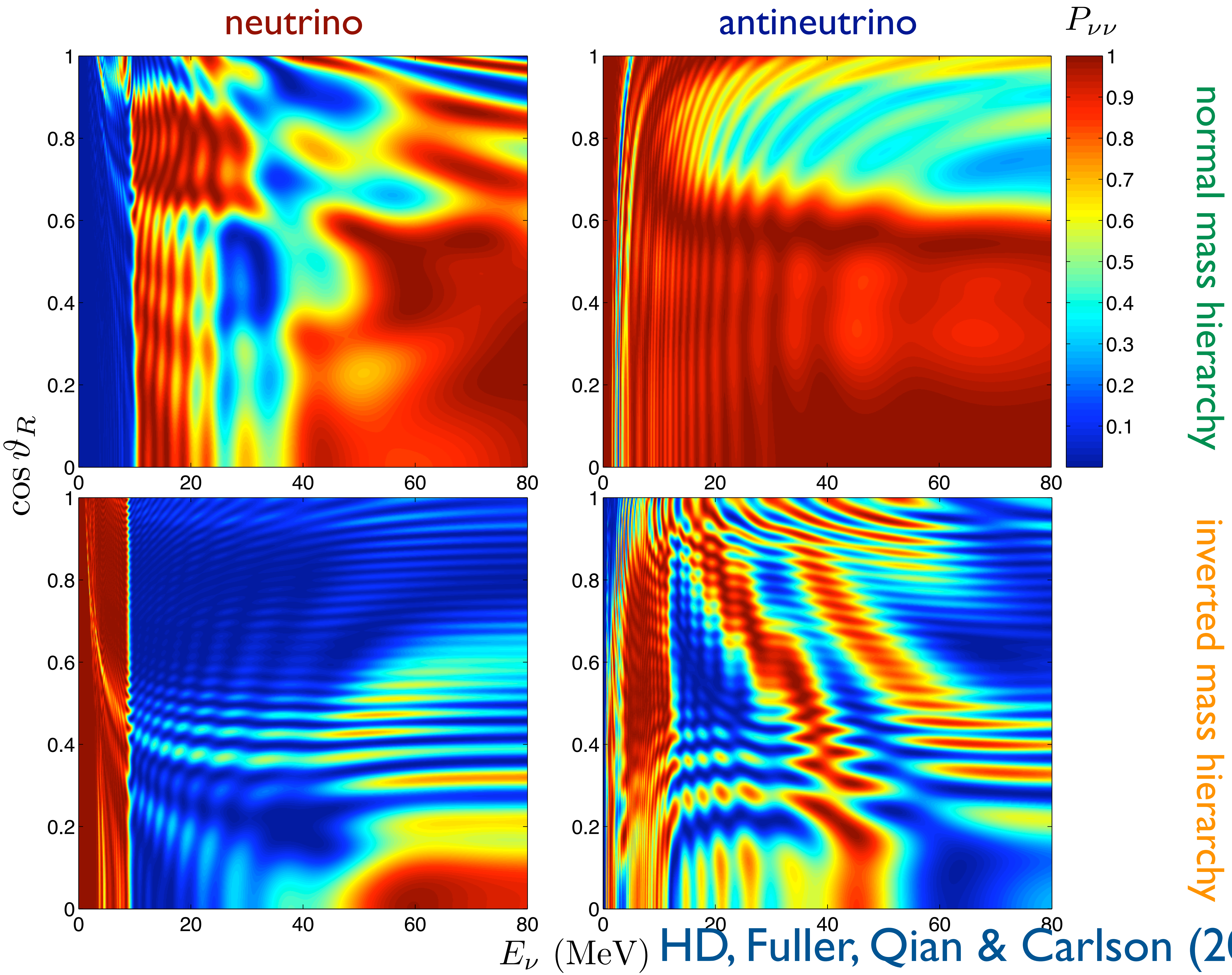


HD, Fuller, Carlson & Qian (2006)

Spectral Swap/Split







HD, Fuller, Qian & Carlson (2006)

Linear Stability Analysis

$$\vec{s}_\omega \longrightarrow \rho_\omega = \begin{bmatrix} s_z & s_x - \mathrm{i}s_y \\ s_x + \mathrm{i}s_y & -s_z \end{bmatrix}$$

$|s_z| \approx 1,\ |s_x| \sim |s_y| \ll 1 \Longrightarrow$ Keep linear terms of $S = s_x - \mathrm{i}s_y$

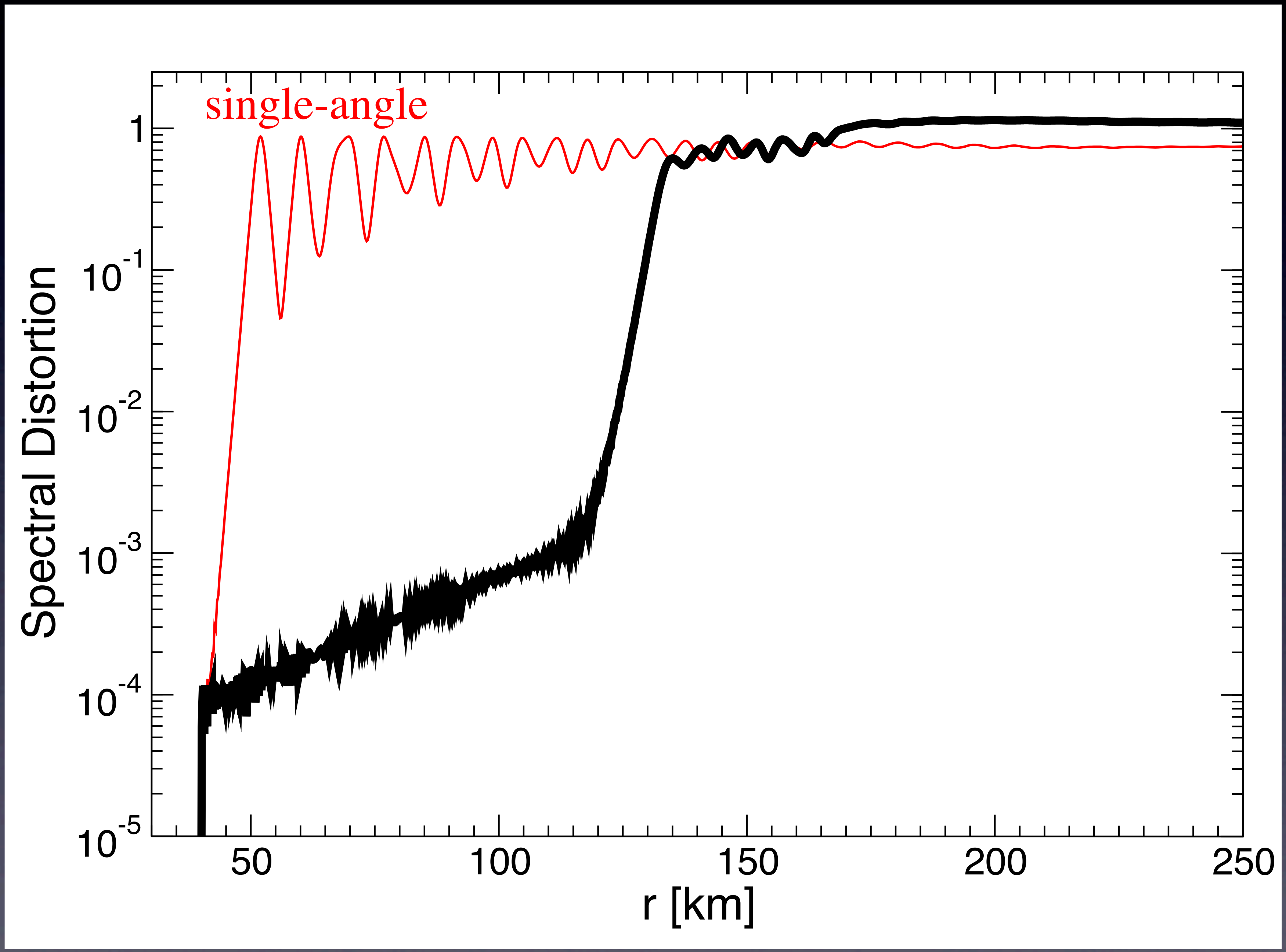
$$\mathrm{i}\dot{S}_\omega \approx \omega S_\omega - \mu \int f_{\omega'} S_{\omega'}\, \mathrm{d}\omega'$$

Pure precession $\Longrightarrow S_\omega \propto e^{-\mathrm{i}\omega_{\mathrm{pr}} t}$

Imaginary ω_{pr} $(= \gamma + \mathrm{i}\kappa) \Longrightarrow$ flavor instability

(Banerjee et al, 2011)

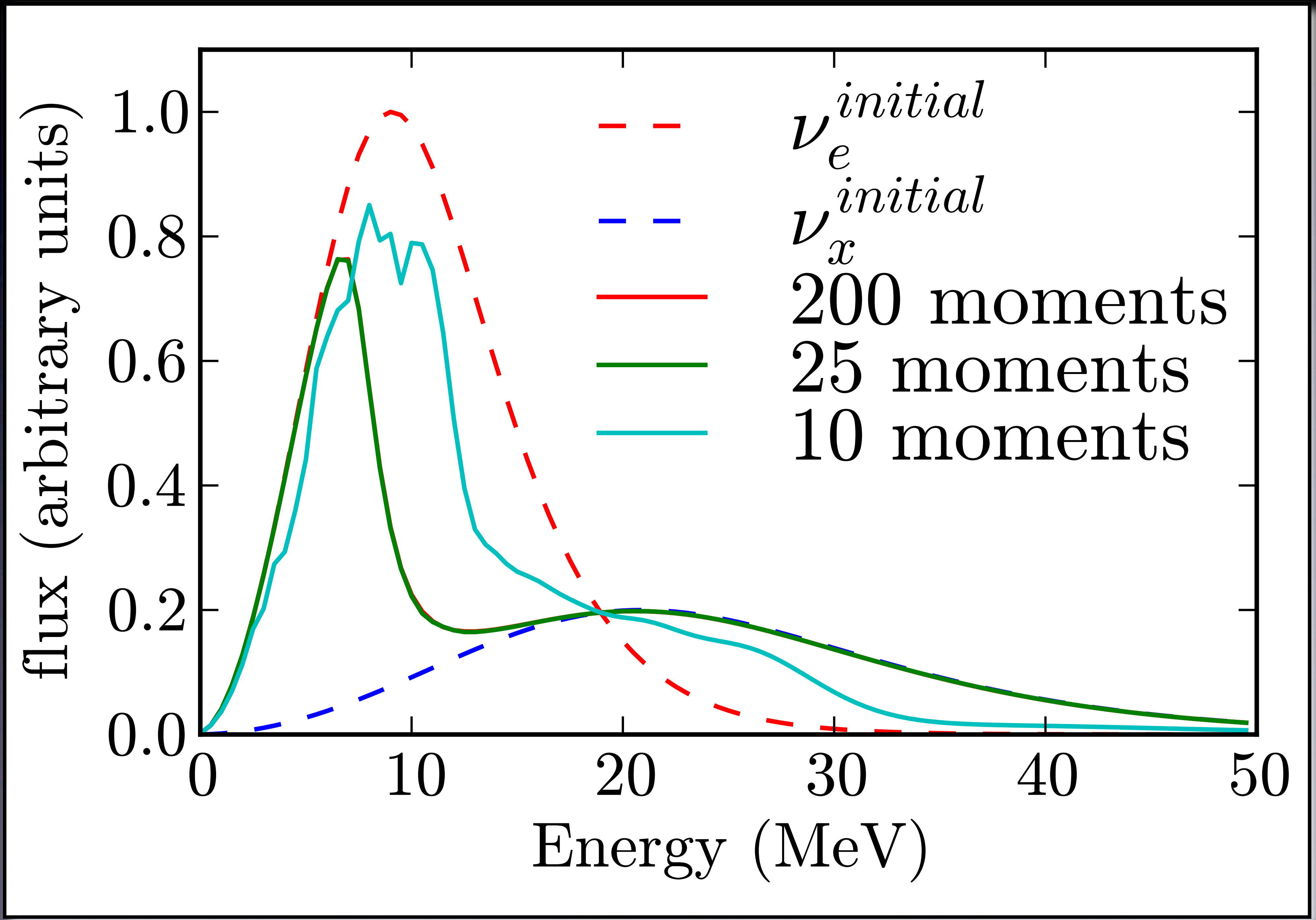
Multiangle Suppression



HD & Friedland (2010)

New Developments and Challenges

Moment Method

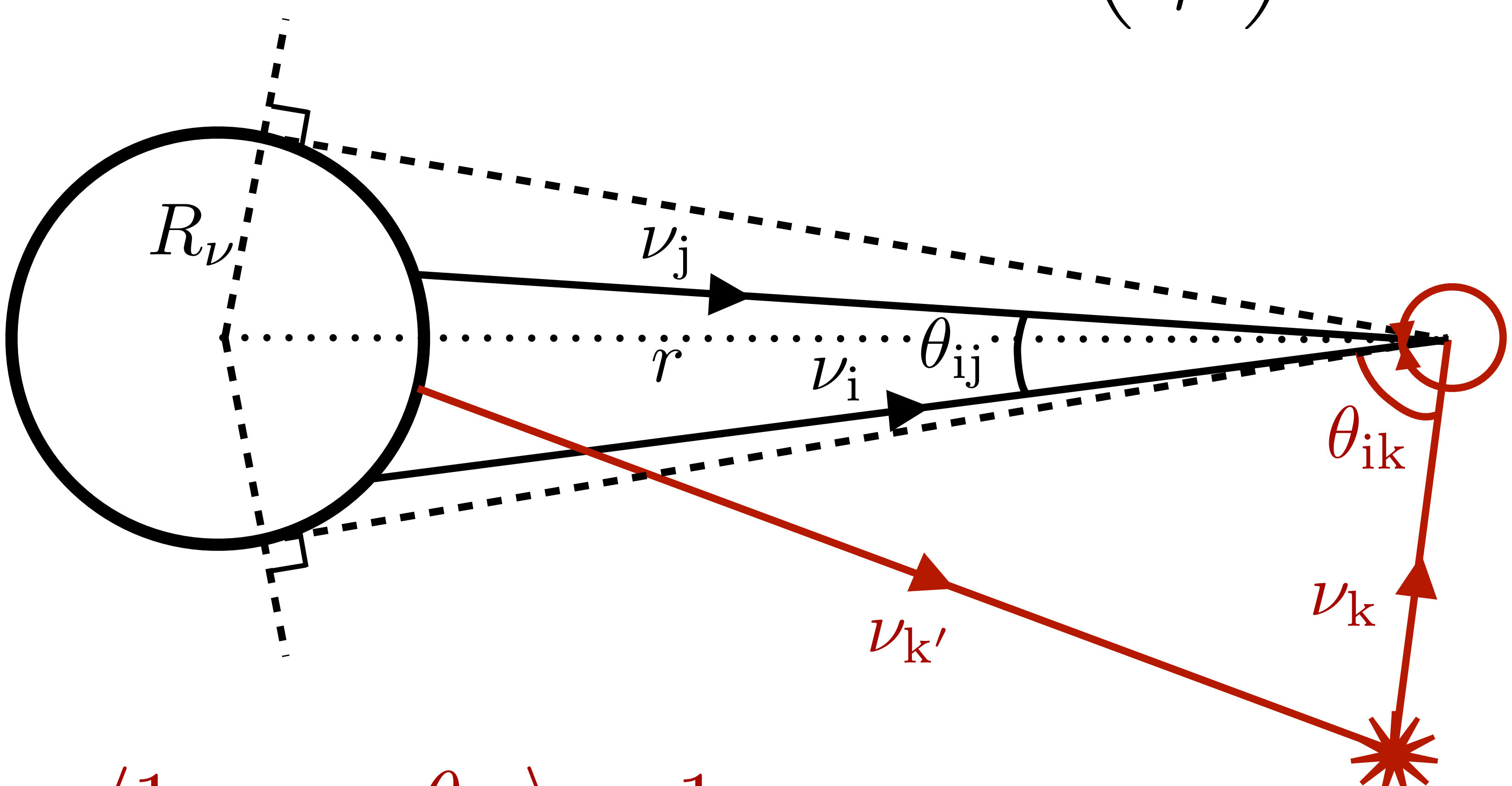


Shalgar & HD (in preparation)

Neutrino Halo

(Cherry et al, 2012)

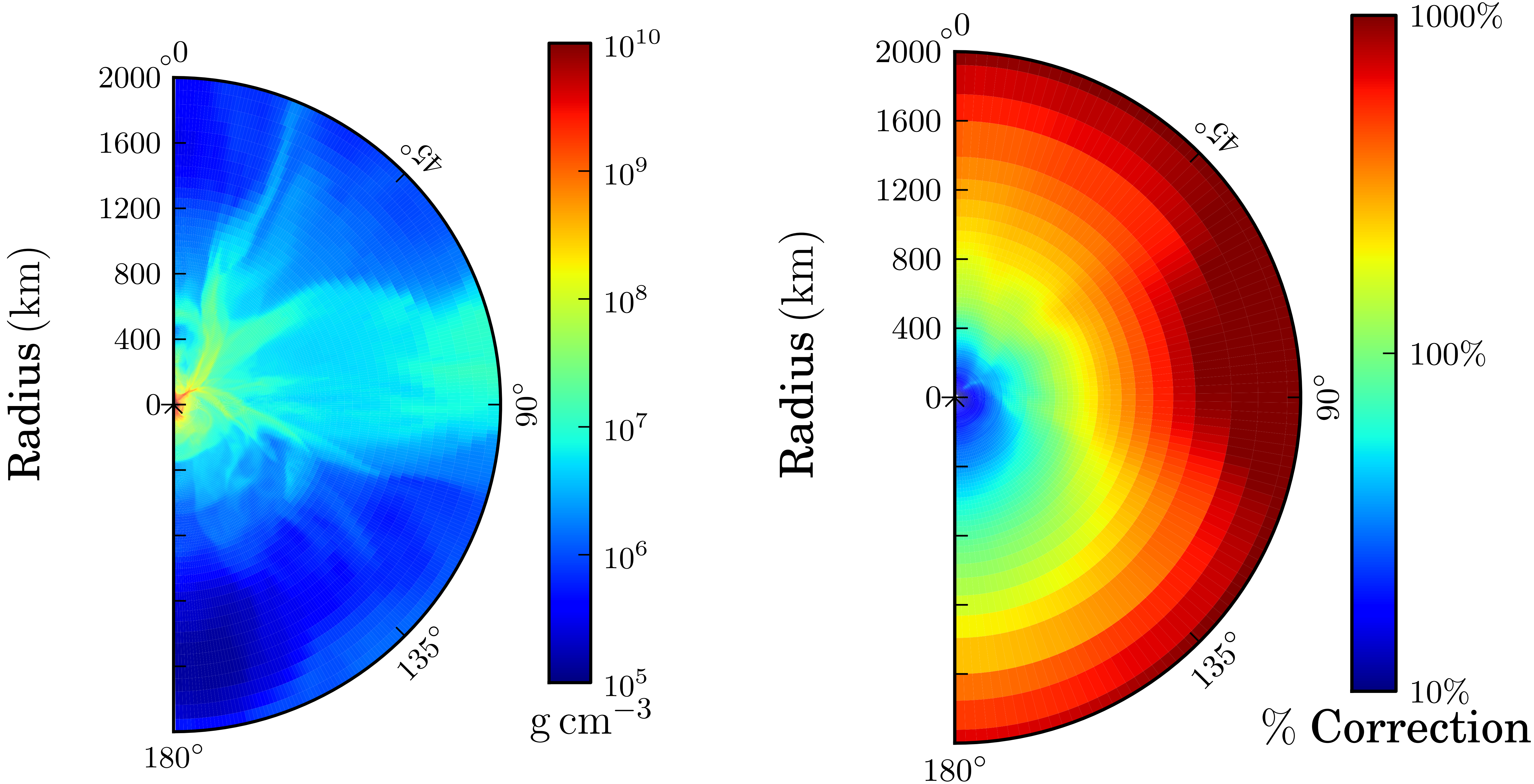
$$r \gg R_\nu \Rightarrow \langle 1 - \cos\theta_{\rm ij} \rangle \propto \left(\frac{R_\nu}{r}\right)^2$$



$$\langle 1 - \cos\theta_{\rm ik} \rangle \approx 1$$

$\sim 10^{-3}$ of all ν's

Neutrino Halo

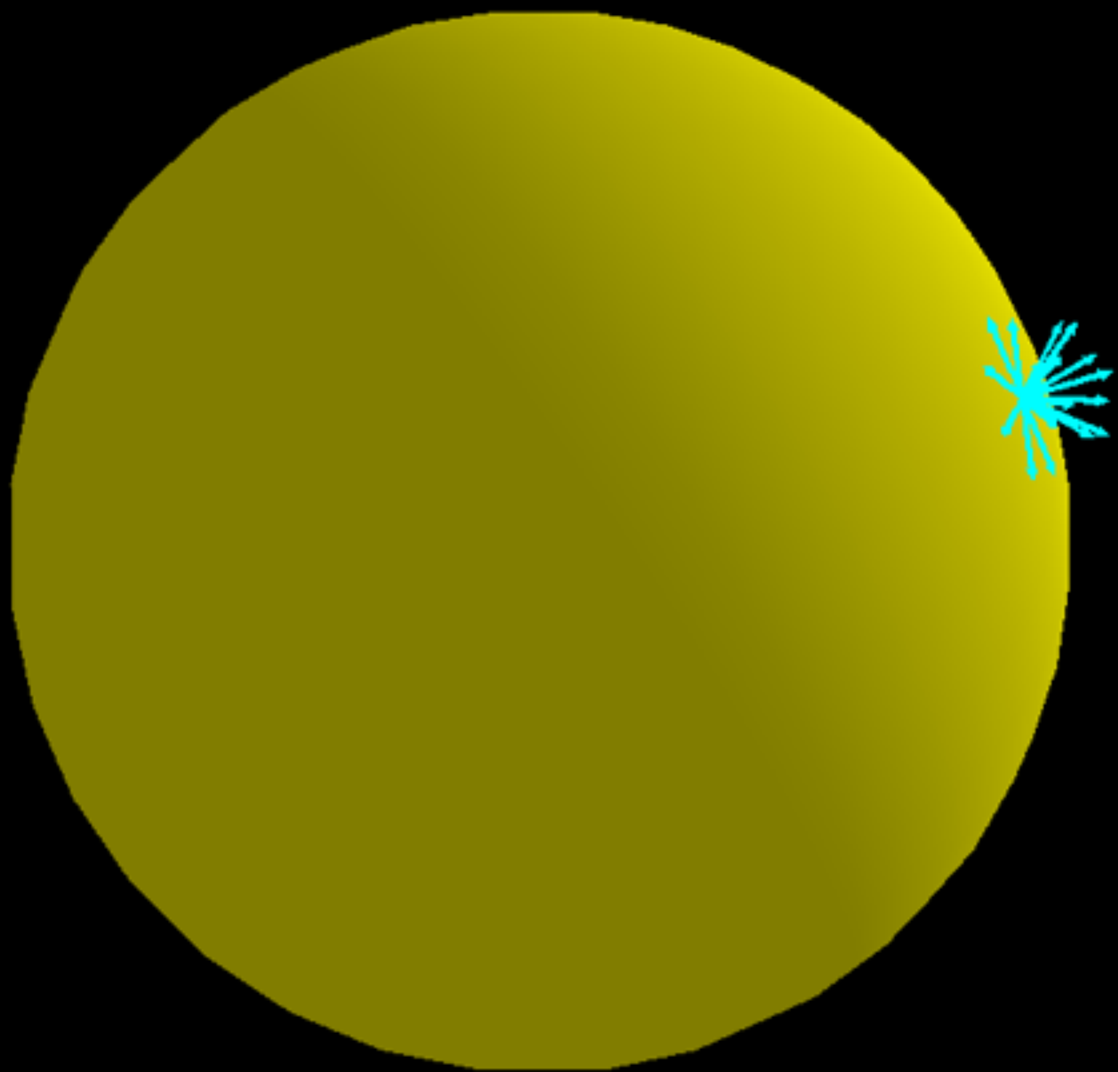


(Cherry et al, 2012)

Spontaneous Symmetry Breaking?

- A symmetry in the EoM does not guarantee that its solution(s) will also be symmetric.
- Even if the system may be approximately symmetric initially, a non-symmetric mode may quickly dominate if it is unstable.
- Numerical calculations suggest that supernova neutrino oscillations may not be axially symmetric even in the (1+2)D model. [Raffelt et al, 2013; Mirizzi, 2013]

(1+3)D



propagation direction

$$\psi(r, E, \vartheta, \varphi)$$

energy

previous assumptions +
Spherical symmetry about the center (Consistency?)

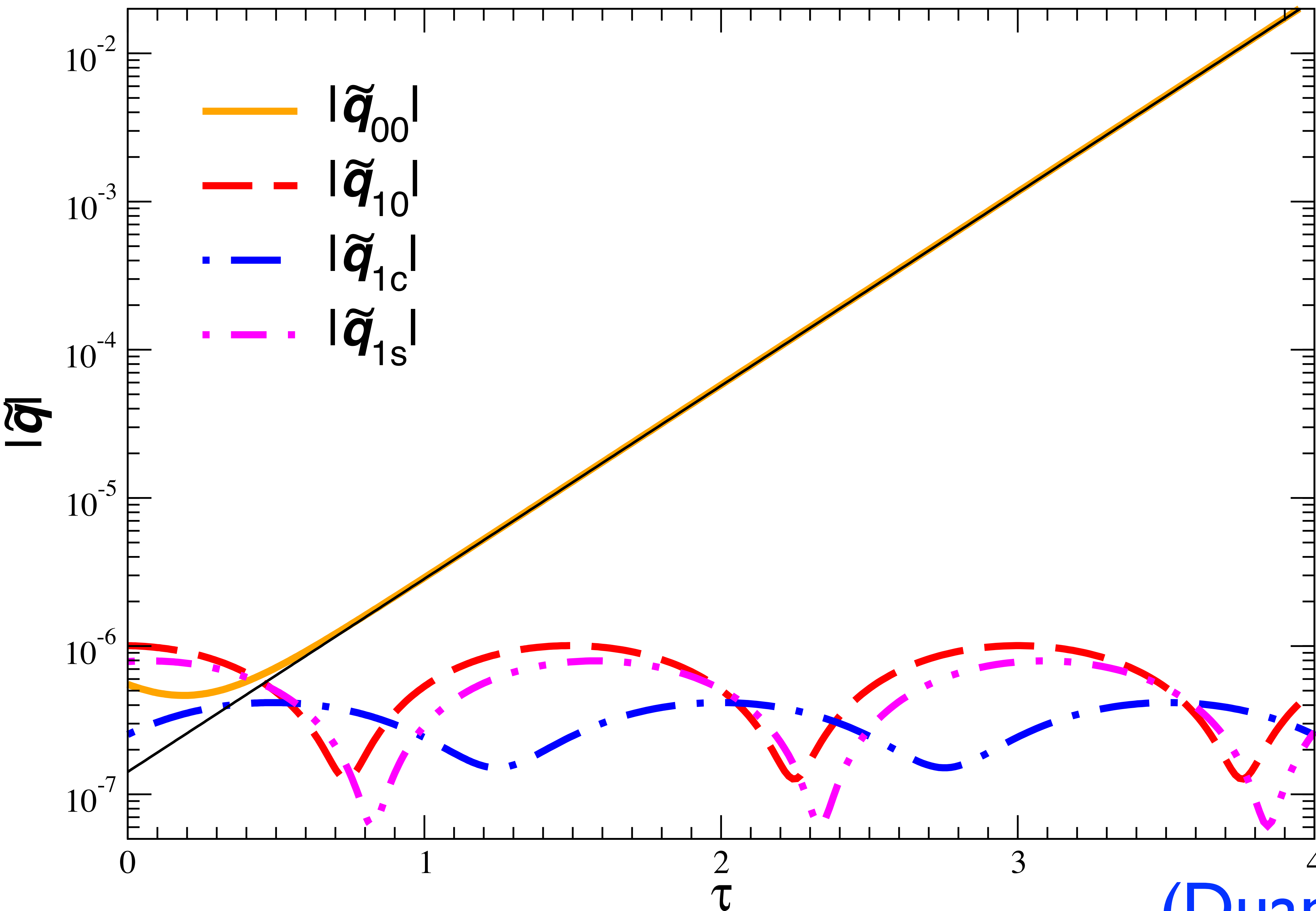
Homogeneous Gas Again

$$1 - \mathbf{p} \cdot \mathbf{p}' = 4\pi \left[Y_{0,0}(\mathbf{p}) Y^*_{0,0}(\mathbf{p}') - \frac{1}{3} \sum_{m=0,\pm 1} Y_{1,m}(\mathbf{p}) Y^*_{1,m}(\mathbf{p}') \right]$$

- Multipole modes are decoupled in the linear Regime
- l=0: μ_{eff}= μ, unstable in IH
- l=1: μ_{eff}= -μ/3 unstable in NH
- l>1: μ_{eff}= 0, always stable

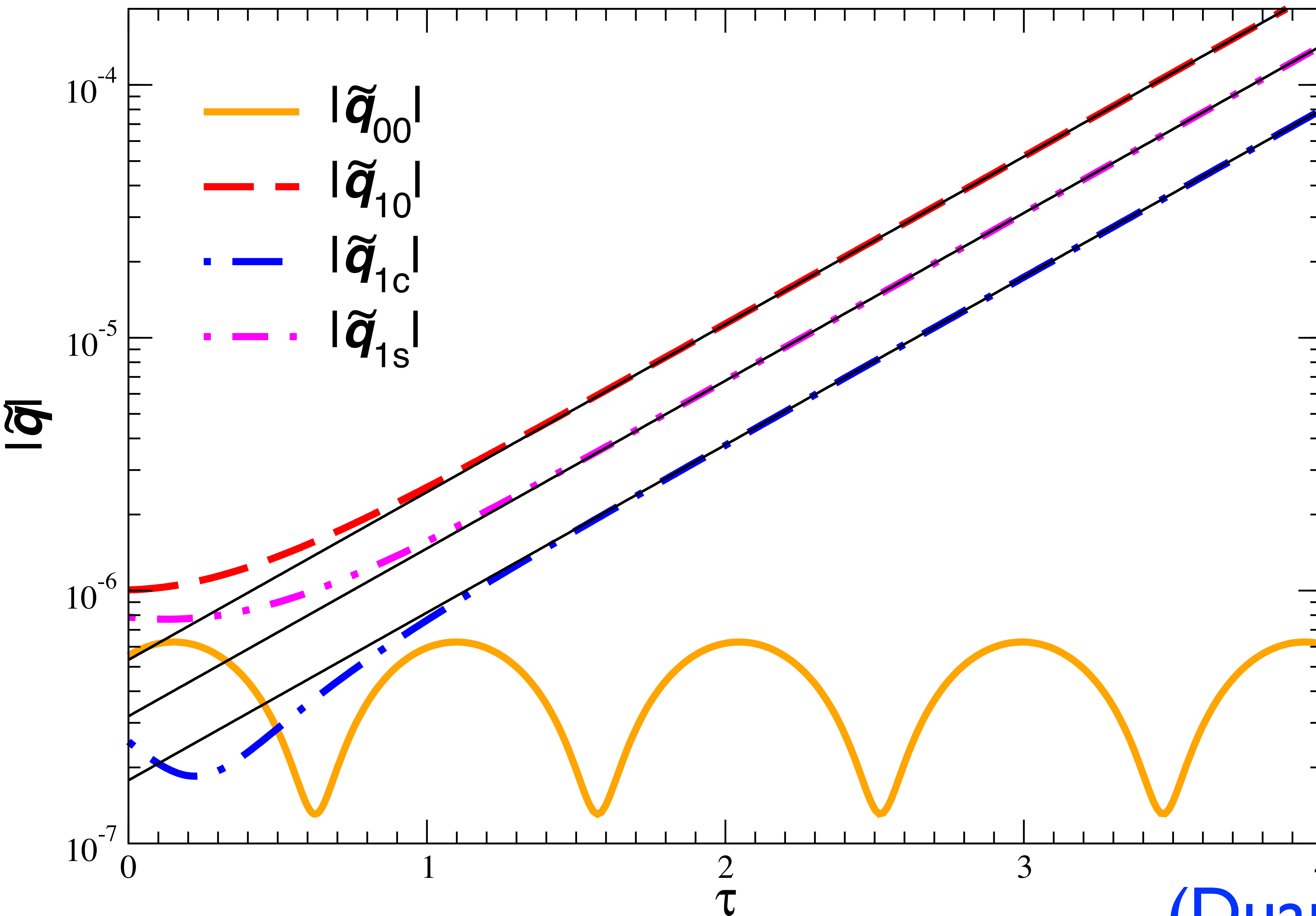
(Duan, 2013)

Inverted Hierarchy



(Duan, 2013)

Normal Hierarchy



(Duan, 2013)

Implications for SN ν

- Collective oscillations can occur in either mass hierarchy.
- Oscillations can occur deeper in the NH case than the IH case.
- The angle-dependent modes break the axial symmetry and the spherical symmetry -- new computing paradigm is needed.

Summary

- Neutrinos offer a unique and direct probe into the center of stars, including supernovae.
- Neutrinos are essential to supernova dynamics and nucleosynthesis.
- Collective neutrino oscillations — a collective quantum phenomenon on the scale of 10 ~100 km?

How do you want do your calculations?