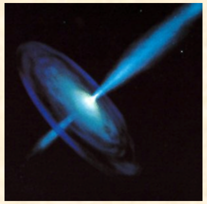
MHD accretion flows

Jim Stone, Yanfei Jiang, Matt Kunz *(Princeton University)*
Shane Davis *(CITA)*
Xuening Bai *(Harvard University)*

Studies of accretion disks in:



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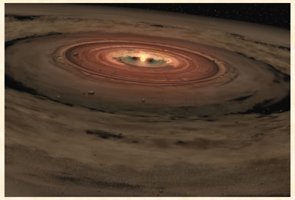


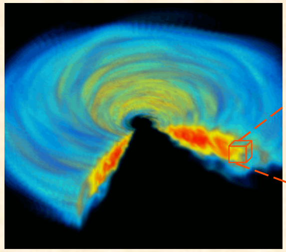
Protostars

Outline

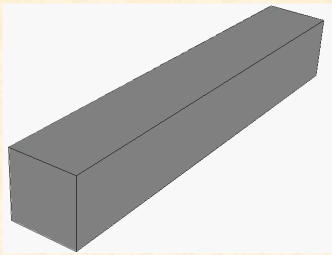
1. Radiation-dominated disks
2. Planetary wakes in MRI turbulence
3. Effect of ambipolar diffusion (AD) on MRI
4. Dust dynamics in protoplanetary disks

MRI Turbulence

Weakly magnetized Keplerian accretion disks are linearly unstable: *magneto-rotational instability (MRI)*



Global simulation



Local simulation

MRI saturates as MHD turbulence.
Turbulence sustained with net flux.
Over last 20 years, hundreds of papers studying nonlinear regime.

1. Radiation Dominated Disks

(work with Yanfei Jiang and Shane Davis)

In black hole accretion disks, radiation pressure exceeds gas pressure inside

$$r/R_G < 170(L/L_{\rm Edd})^{16/21}(M/M_\odot)^{2/21}$$

(Shakura & Sunyaev 1973). Radiation needs to be included in dynamical models.

If stress $\tau_{r\phi} = \alpha P$ then radiation dominated disks are subject to both

- Viscous instability (Lightman & Eardley 1974)
- Thermal instability (Shakura & Sunyaev 1976)

Still not clear if such instabilities really exist with MRI.

Method: Equations of radiation MHD

Euler equations + Maxwell's equations + zeroth and first moment equations.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{vv} + P + B^2/2 - \mathbf{BB}) = -\mathbb{P}\mathbf{S_M}$$

$$\frac{\partial E}{\partial t} + \nabla \cdot [(E+P)\mathbf{v} + (B^2/2)\mathbf{v} - \mathbf{B}(\mathbf{B}\cdot\mathbf{v})] = -\mathbb{PC}S_E$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0$$

$$\frac{\partial E_r}{\partial t} + \mathbb{C}\nabla \cdot \mathbf{F}_r = \mathbb{C}S_E$$

$$\frac{\partial \mathbf{F}_r}{\partial t} + \mathbb{C}\nabla \cdot \mathsf{P}_r = \mathbb{C}\mathbf{S_M}$$

E_r, F_r, P_r are radiation energy density, flux, pressure in Eulerian (fixed) frame.

Source terms are O(v/c) expansion of material-radiation interaction terms in fluid frame.

Lowrie et al 1999

$$\mathbf{S_M} = -(\sigma_a + \sigma_s)\left(\mathbf{F}_r - \frac{\mathbf{v}E_r + \mathbf{v}\cdot\mathsf{P}_r}{\mathbb{C}}\right) + \frac{\mathbf{v}}{\mathbb{C}}(\sigma_a T^4 - \sigma_a E_r)$$

$$S_E = (\sigma_a T^4 - \sigma_a E_r) + (\sigma_a - \sigma_s)\frac{\mathbf{v}}{\mathbb{C}} \cdot \left(\mathbf{F}_r - \frac{\mathbf{v}E_r + \mathbf{v}\cdot\mathsf{P}_r}{\mathbb{C}}\right)$$

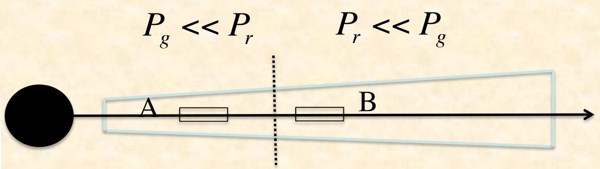
Towards Better Numerical Methods

The three challenges of numerical radiation MHD:

- Need closure relation **P=fE**
  - Compute variable Eddington tensor (VET) from formal solution of time-independent transfer equation.
- Source terms can be very stiff
  - Use modified Godunov method
- Wide range of timescales associated with v, C_s, c
  - Requires fully implicit (backward Euler) differencing of radiation moment equations

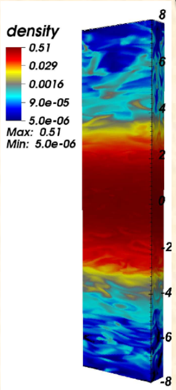
Each of these three ingredients are implemented as a radiation module in the Athena MHD code. Davis, Stone, & Jiang 2012
Jiang, Stone, & Davis 2012

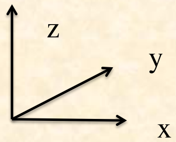
Local shearing-box simulations of radiation dominated accretion disks



Electron scattering dominates.

Free-free absorption dominates.





Shearing box

Box size: Hx4Hx16H
Resolution: 32/H – 64/H

Parameters

Radiation pressure dominated regime

Table 1:: Location A

Parameters	Value	Comment
M	$6.62 M_\odot$	Mass of Central Black Hole
r	$30\,(GM/c^2)$	Radius
ρ_0	5.66×10^{-2} g cm^{-3}	Initial Mid-plane density
T_0	2.45×10^7 K	Initial Mid-plane temperature
τ	3.514×10^4	Total Electron Scattering Optical Depth

Gas pressure dominated regime

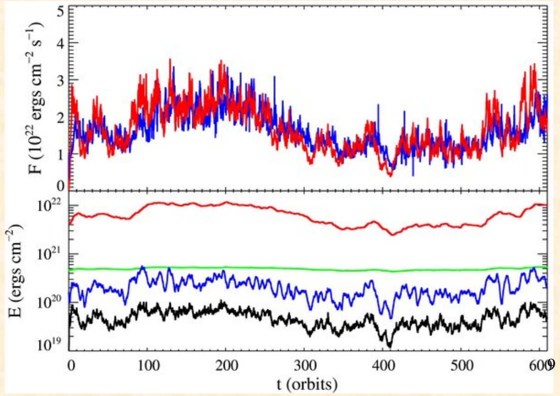
Table 2:: Location B

Parameters	Value	Comment
M	$6.62 M_\odot$	Mass of Central Black Hole
r	$300\,(GM/c^2)$	Radius
ρ_0	1.12×10^{-2} g cm^{-3}	Initial Mid-plane density
T_0	2.89×10^6 K	Initial Mid-plane temperature
τ	1.06×10^4	Total Electron Scattering Optical Depth

Previous results using FLD

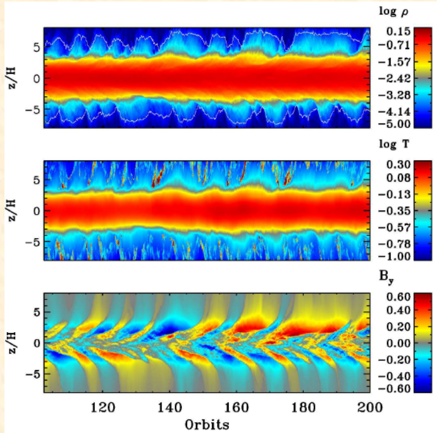
Same setup and parameters as Hirose et al. (2006; 2009), except we use a *much larger computational domain*.
They found radiation pressure dominated disks are stable in using FLD module in the ZEUS code.

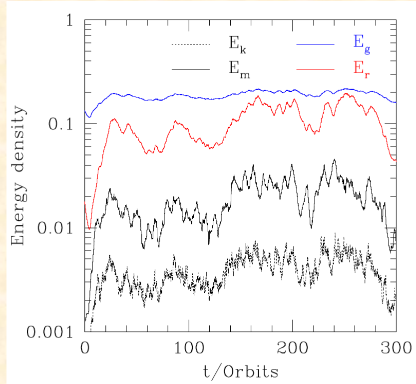
H/4 x 2H x 4H domain



B: gas pressure dominated case

Space-time diagram

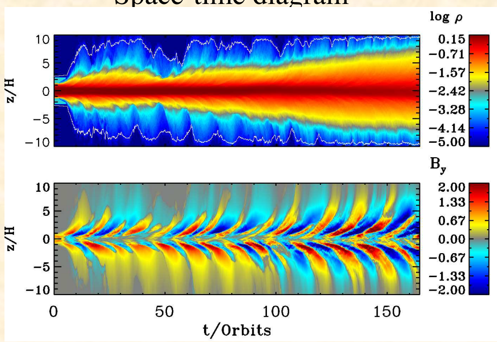


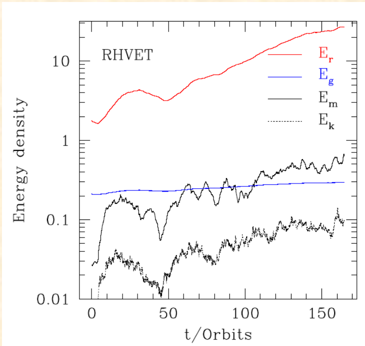


Stable solution

A: radiation dominated case ($P_{rad}/P_{gas} = 4.13$)

Space-time diagram

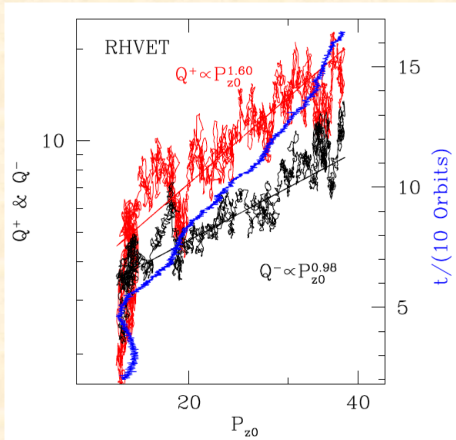




Thermal runaway: energy increases with time.

Stop calculation when photosphere hits boundary.
Restart with taller box and energy continues to increase…

Heating and Cooling Rates



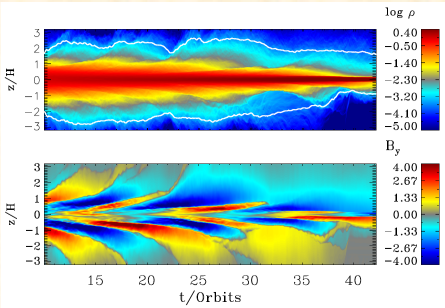
Instability criterion:

$$\left.\frac{\partial \log Q^+}{\partial \log P_t}\right|_\Sigma > \left.\frac{\partial \log Q^-}{\partial \log P_t}\right|_\Sigma$$

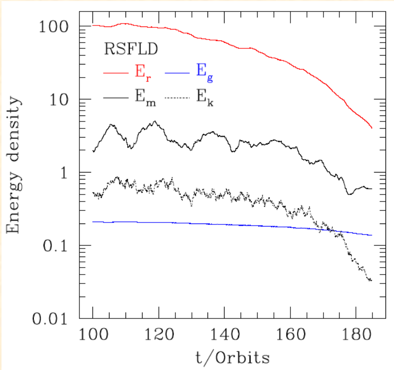
Instability criterion satisfied, however the exact scaling differs from standard α model.

Lower surface density (P_{rad}/P_{gas}=206)

Space-time diagram



Calculation with VET:
Thermal runaway; disk collapses



Calculation with FLD in Athena:
Thermal runaway; disk collapses

Must stop both calculations when disk is too thin to be resolved.

Summary

1. Radiation-dominated disks

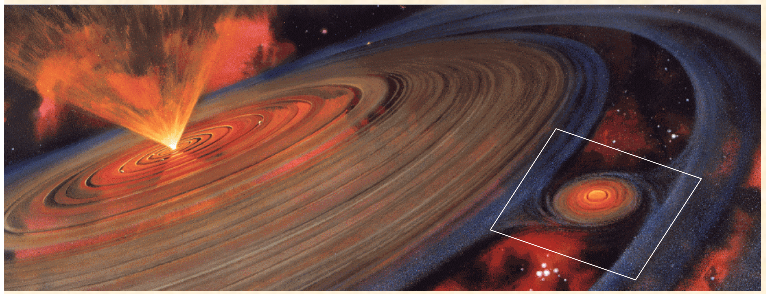
   Strongly radiation-dominated disks are thermally unstable

2. Planet wakes in MHD turbulence.

(work with Zhaohuan Zhu)

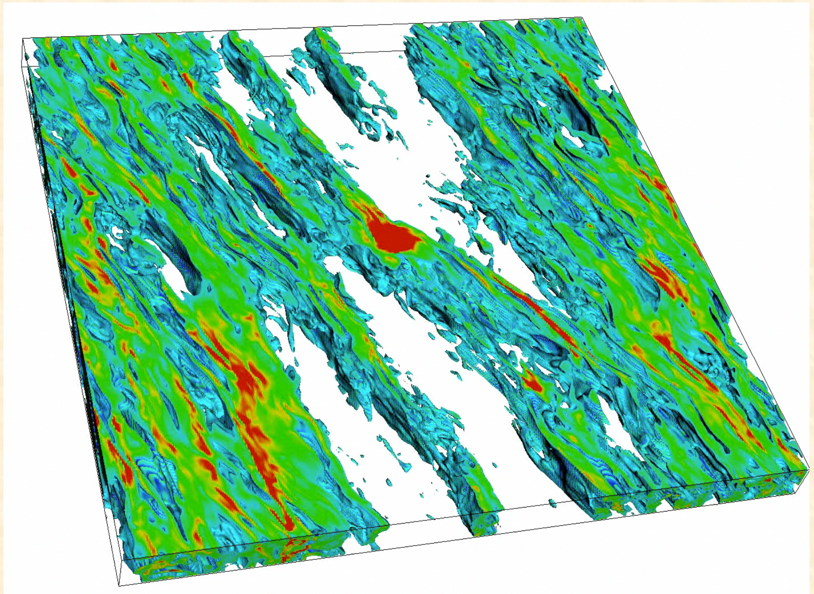
Interaction between planet and disk launches density waves, opens a gap. Torque from gas can make planet migrate in disk.

Simulations required to study properties of gap and migration.



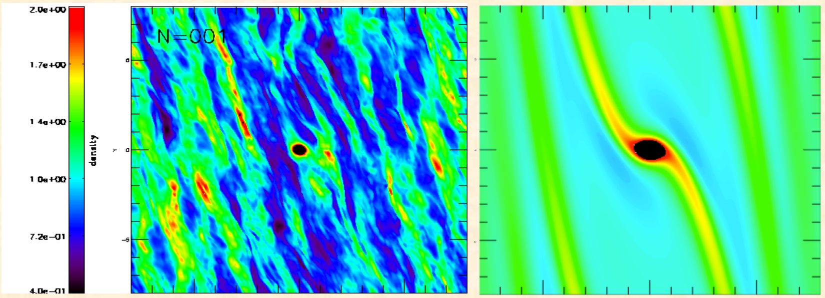
Local shearing box simulations of planet in disk.

Density isocontours: thermal mass ($M \sim 0.1 M_J$) planet in MRI turbulence



Comparison of wake.

Plots of surface density

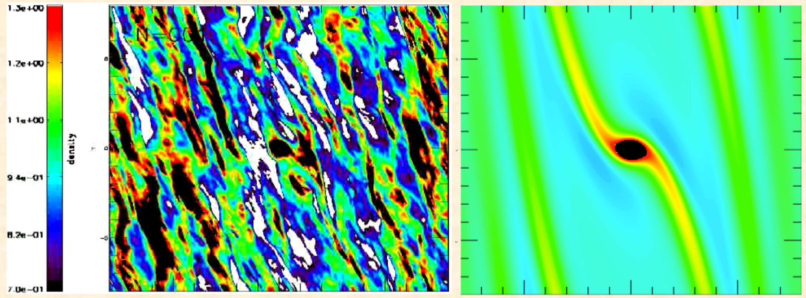


Turbulent MHD disk

Viscous hydrodynamic disk
Same α as MHD

Comparison of wake.

Plots of surface density



Turbulent MHD disk

Viscous Hydrodynamic disk

Time averaging reveals wake!

Profiles of gap and wake substantially different than α-disk model.
Representing turbulence by viscosity not valid for this problem.

Summary

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2. Planetary wakes in MRI turbulence

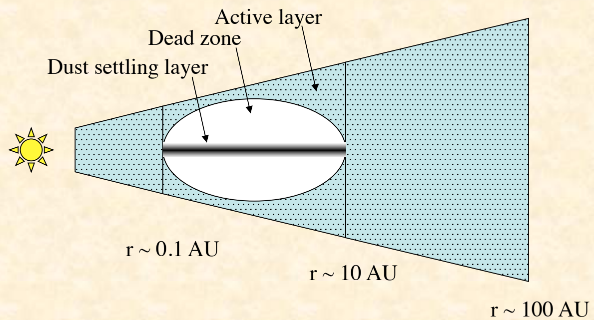
   Cannot represent all effects of turbulence as diffusivity.

3. Effect of AD on MRI

(work with Xuening Bai)

It seems likely PPDs will have a layered structure: Gammie 1996, Glassgold et al. 1997

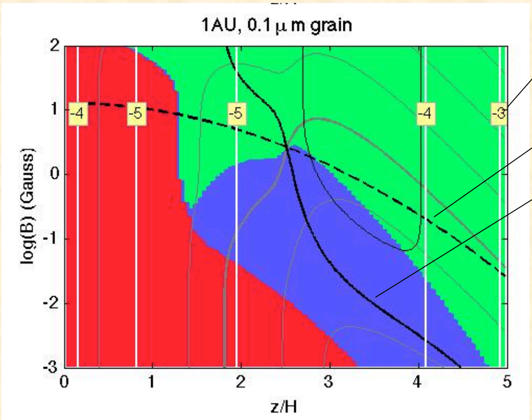
1. Inner region: well ionized and turbulent.
2. Central region:
   - turbulent active layers, MRI controlled by non-ideal MHD.
   - laminar dead zone.
3. Outer region: turbulent, with MRI controlled by AD.



Which terms dominate in a proto-stellar disk?

Must adopt a disk model:

- Minimum-mass solar nebula, $\Sigma = \Sigma_0 r^{-3/2}$, with $\Sigma_0 = 1700$ g/cm^2
- Non-thermal source of ionization (e.g. X rays)
- multi-species ion chemistry model including grains.



Log(recombination time) normalized by Ω^{-1}

β=1

Elsasser number = 1

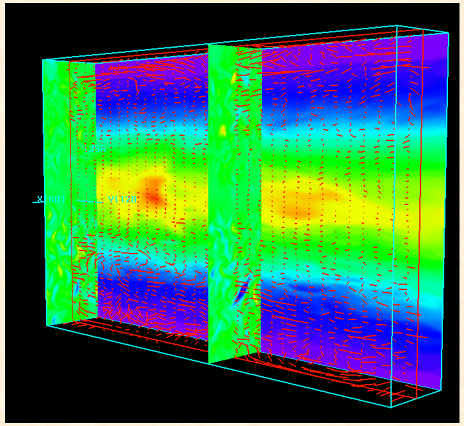
Red = Ohmic
Blue = Hall
Green = AD

Salmeron & Wardle 2007; Wardle 2007

Layered disks with AD

(Gammie 1996; Igea & Glassgold 1998)

- Studied using 3-D simulations of stratified disks with non-uniform resistivity $\eta = \eta(z)$ ***and AD*** (Bai & Stone 2013)



Low η, Large AD (active layer)

High η, Large AD (dead zone)

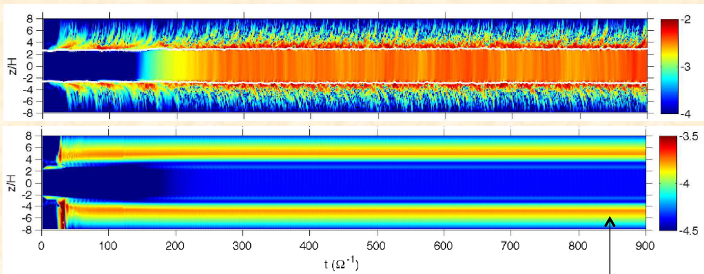
Low η, Large AD (active layer)

Vertical flux + AD in active layer produces powerful winds

Space-time plots of Maxwell stress

Only Ohmic

Ohmic + AD

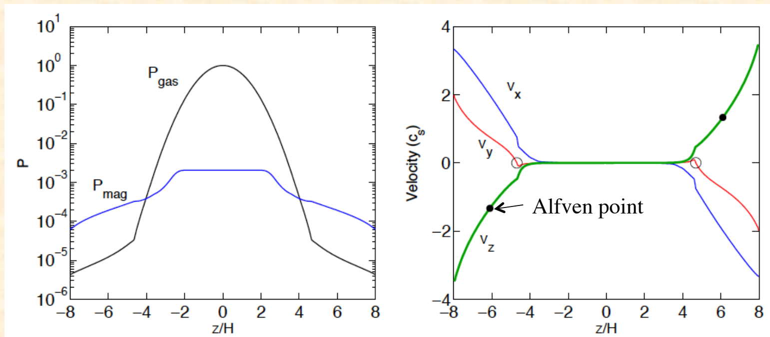


No turbulence, but steady wind!

Negative feedback: MRI amplifies B, increasing effect of AD

With AD: super-Alfvenic winds

Time-averaged, vertical profiles in wind solution



Details in remarkable agreement with steady-state wind theory.

With AD, qualitative change in solution.

Need *global* simulations with Ohmic + AD + Hall to confirm.

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3. Effect of ambipolar diffusion (AD) on MRI

   AD is important for MRI in PPDs, produces wind

4. Dust Dynamics in Protoplanetary Disks

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1. Inner region: well ionized and turbulent.
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   - laminar dead zone. Dust may settle in dead zone.
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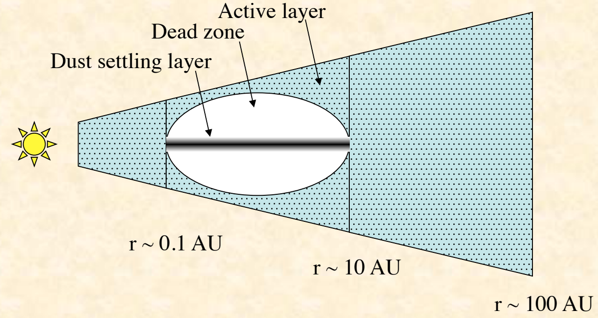


Particle aerodynamics

Weidenschilling 1977
Cuzzi et al. 1993

Eqns. of motion for particles in disk: $$\frac{d\mathbf{v}_i}{dt} = -\Omega_K^2 \mathbf{r} + \underbrace{\frac{\mathbf{v}_g - \mathbf{v}_i}{t_s}}_{\text{gas drag}}$$

Define a *dimensionless stopping time* $\boxed{\tau_s = \Omega t_s}$

For small particles $a \leq \lambda_{\rm mfp}$ drag force given by the Epstein regime:

$$\tau_s = \frac{\rho_s a \Omega_K}{\rho_g c_s} = 4.41 \times 10^{-3} \left(\frac{\rho_s}{3 \text{ g cm}^{-3}}\right) \left(\frac{a}{\rm cm}\right) r_{\rm AU}^{3/2}$$

Particles must be cm or larger for $\tau_s > 10^{-3}$

Radial drift of particles

Nakagawa, Sekiya & Hayashi 1986 (NSH)

- **Gas** in the disk orbits at slightly sub-Keplerian velocity due to small outward pressure gradient.
- **Particles** do not feel pressure, orbit at V_K

This leads to a small difference in the angular velocity of particles and gas: $$\boxed{\Delta V_\phi = \frac{\eta V_K \tau_s^2}{(1+\epsilon)^2 + \tau_s^2}}$$

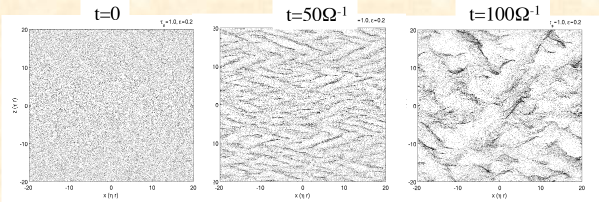
where $\eta \equiv -\frac{1}{2\rho_g V_K^2}\frac{\partial P}{\partial \ln r} \sim \left(\frac{c_s}{V_K}\right)^2$ $\quad \epsilon \equiv \rho_d/\rho_g$

Drag between the gas and particles generates a torque which causes particles to spiral inwards, and gas to drift slowly outwards. Radial drift speed is: $$\boxed{\Delta V_r = \frac{-2(1+\epsilon)\eta V_K \tau_s}{(1+\epsilon)^2 + \tau_s^2}}$$

Lifetime of ~1 m sized particles at 1 AU is only ~200 orbits

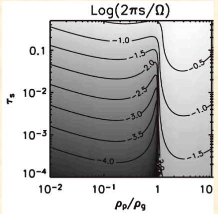
Streaming Instability (SI)

Momentum feedback *on the gas from the particles* causes a linear instability, driven by the radial drift velocity.



Drives clumping of particles, low Mach number turbulence in the gas.
Typical length scale of instability ~ $(\eta V_K/c_s)H = 0.05H$ for MMSN

Growth rate s ~ Ω;
Largest for $\tau_s \sim 1$ and $\varepsilon \sim 1$.



Youdin & Goodman 2005
Youdin & Johansen 2007
Johansen & Youdin 2007

Numerical algorithms: particles.

Bai & Stone 2010

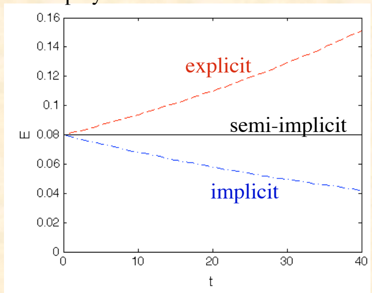
Arbitrary number of super-particles, with arbitrary mass distribution.

Integrate equation of motions for particles using second-order *semi-implicit* integrator which conserves orbital energy exactly. *Fully implicit* integrator is used for particles with very short stopping time.

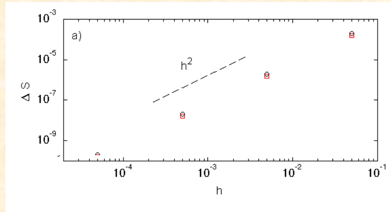
"Triangular-shaped cloud" (TSC) interpolation between grid and particle positions.

Fully conservative, parallelized with MPI.

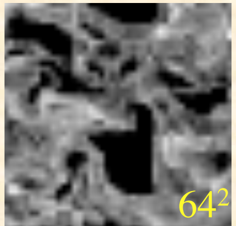
Energy evolution for single-particle epicyclic motion test.

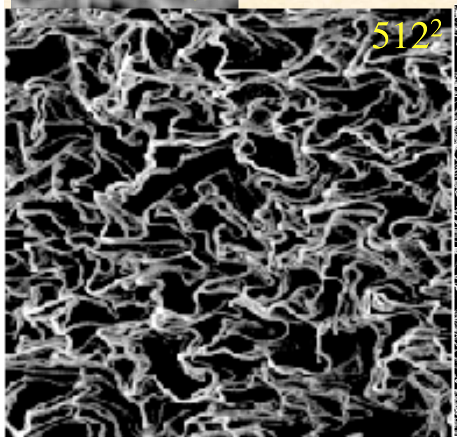


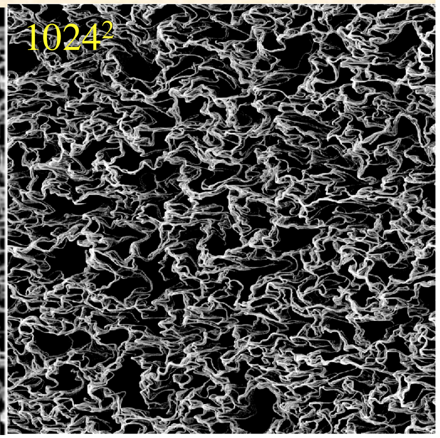
Second-order convergence of errors in single-particle integration.



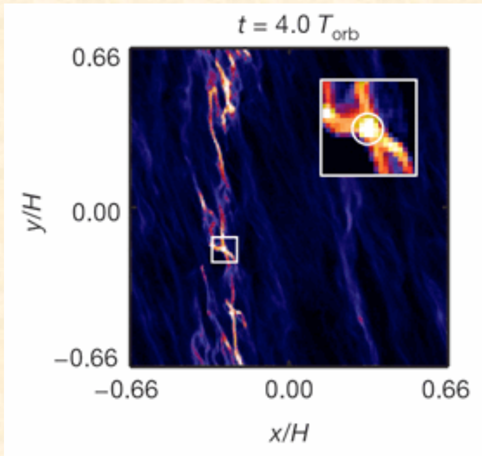
Convergence of τ_s=0.1, ε=1.0







With self-gravity, clumps formed by SI can collapse into planetesimals



Simulation of SI+gravity by Johansen et al. 2007

BUT: how do you get to cm sized particles so SI can operate?

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   Cannot represent all effects of turbulence as diffusivity.

3. Effect of ambipolar diffusion (AD) on MRI

   AD is important for MRI in PPDs, produces wind

4. Particles in Disks

   Streaming Instability can produce seeds for planetesimals